Quadratic Polynomial Form of Electric Arc Furnace Equation

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Abstract

Purpose – Electric arc furnaces are very often modelled using combined models which cover separately deterministic and stochastic phenomena taking place in the furnace. The deterministic part is expressed by nonlinear differential equations. A closed form of the solution to one of the most popular nonlinear differential equations used for the AC electric arc modelling does not exist for some values of the parameters. The paper goal was to convert electric arc furnace equation for these parameters to the quadratic polynomial form which significantly simplifies solution.

Design/methodology/approach – The solution has been obtained in the time domain by a sequence of transformations of the original nonlinear equation which lead to a system of quadratic equations, for which a periodic solution can be found easily using harmonic balance method (HBM).

Findings – Quadratic polynomial form of electric arc furnace nonlinear equation in the case for which the solution to the nonlinear differential equation describing electric arc cannot be obtained in a closed form.

Research limitations/implications – The complete model of the arc requires extension of the deterministic solution obtained for the quadratic polynomial form using stochastic or chaotic component.

Practical implications – The obtained results simplify determination of the arc voltage or radius time waveforms if a closed form solution does not exist. The arc model can be used to evaluate the impact of arc furnaces on power quality during the planning stage of new plants. The proposed approach facilitates calculation of the arc characteristic.

Originality/value – In order to avoid problems occurring when a large number of harmonics is required or the system contains strong nonlinearities, a transformation of the original equation has been proposed. Nonlinearities present in the equation have been transformed into purely quadratic polynomial terms. It facilitates application of the classical HBM and allows to follow periodic solutions of the arc equation when its parameters are varied. It also enables better understanding of the phenomenon described by the equation and makes easier the extension of the arc model in order to cover the time-varying character of the arc waveforms.

Keywords – electric arc furnace, nonlinear differential equations, quadratic polynomial form, nonlinear loads

Paper type – Research paper

1. Introduction

Understanding properties of solutions of differential equations describing physical phenomena is fundamental for modern engineering. A differential equation describing the AC electric arc is just one of the examples. Electric arc furnaces are commonly used for melting metals in steel industry. Unfortunately, nonlinear characteristic of the arc furnace and its stochastic behavior are the cause of voltage flicker and waveform distortions in power systems (Manchur and Erven 1992; Alves *et al.*, 2010; Gomez et al., 2010).

The stochastic nature of processes which take place in the furnace makes the development of a realistic arc model a challenging task. However, a reliable model is required to estimate the degradation of the power quality caused by the arc furnace and to take some actions which enable to overcome the mentioned above negative effects.

During melting the arc extinguishes and starts again in a random way. In spite of the arc stochastic nature, its analysis usually starts using deterministic characteristic and the time-varying nature is taken into account in the second step. It is made with the aid of chaotic, stochastic or

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modulated components which are added to the deterministic solution (Ozgun and Abur, 1999; Golkar and Meschi, 2008; Alves *et al.*, 2010; Gomez et al., 2010).

This paper is a continuation of other works (Grabowski and Walczak, 2011; Grabowski and Walczak, 2013) in which a closed form solution of one of the most popular deterministic nonlinear differential equations used for the arc modeling (Acha et al., 1990) has been developed for some arc parameters. It seems that a closed form solution for the other arc parameters does not exists (Zwillinger, 1997). However, in this case periodic solutions can be found by combining the harmonic balance method (HBM) and a continuation method (Cochelina and Vergez, 2009). A transformation of the nonlinearities present in the equation into purely quadratic polynomial terms has been proposed in the paper. It makes the determination of periodic solutions easier.

The future task consists in considering some parameters of the arc as stochastic variables to obtain a realistic arc model. This task will be for sure easier to accomplish if the closed form solutions or periodic solutions are analyzed instead of using the numerical approach.

2. Arc furnace model

Deterministic and stochastic components can be observed in voltage-current (V-I) arc characteristics. The share of both components depends on the phase of the melting process (Gomez et al., 2010).

Modeling of the deterministic V-I characteristic can be made with the help of nonlinear differential equations (Gomez et al., 2010; Ozgun and Abur, 1999; Acha et al., 1990), piece-wise linear approximations (Golkar and Meschi, 2008), mixed approximations using exponential and linear functions (Golkar and Meschi, 2008), approximations using shifted and amplified step function (Wang Yongning et al., 2004). The paper follows the first mentioned above approach. The following nonlinear differential equation describing a single-phase electric arc can be derived from the power balance equation (Acha et al., 1990):

$$k_1 r^n(t) + k_2 r(t)\frac{dr(t)}{dt} = \frac{k_3}{r^{m+2}(t)} i^2(t)\,, \tag{1}$$

where:

$r(t)$ – the arc radius,
$i(t)$ – the arc current,
k_j – the proportionality constants, j = 1, 2, 3,
n, m – the arc parameters, n = 0, 1, 2, m = 0, 1, 2.

The current waveform $i(t)$ is treated as an input data when analyzing the arc phenomenon. Equation (1) can be also used to get the characteristic of a three-phase electric arc (Gomez et al., 2010).

The general arc equation (1) is a first-order nonlinear differential equation. Unfortunately, there are no general analytical methods which allow to solve any nonlinear differential equation, even of the first-order. The general arc equation expressed by (1) is nor separable neither exact, so there is no simple way to find its solution in a closed form (Zwillinger, 1997).

Application of a substitution given in (Grabowski and Walczak, 2013) allows to convert (1) into the following first-order differential equation:

$$\frac{dy(t)}{dt} = \alpha\, y^k(t) + f(t)\,, \tag{2}$$

where:

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$$f(t) = \frac{(m+4)\,k_3}{k_2}\, i^2(t)\,, \tag{3}$$

$$\alpha = -\frac{(m+4)\,k_1}{k_2}\,,\; k = \frac{n+m+2}{m+4}\,. \tag{4}$$

For n = 2 and m = 0, 1 or 2 the coefficient k is equal to 1 and the linear first-order differential equation is obtained. A closed form solution of such equation is well known (Zwillinger, 1997). On the base of this solution the arc radius $r(t)$ and subsequently the arc voltage $u(t)$ and conductance $g(t)$ can be determined (Acha et al., 1990):

$$r(t) = y^{\frac{1}{m+4}}(t)\,, \tag{5}$$

$$u(t) = \frac{k_3}{r^{m+2}(t)}\, i(t)\,, \tag{6}$$

$$g(t) = \frac{r^{m+2}(t)}{k_3}\,. \tag{7}$$

For n = 0 and m = 0, 1, 2 the coefficient k is equal to 1/2, 3/5 and 2/3. For n = 1 and m = 0, 1 or 2 it is equal to 3/4, 4/5 and 5/6. In both cases, the HBM may be applied to obtain a periodic solution with fundamental angular frequency ω_0, which is equal to the angular frequency of the exciting term, i.e. the current $i(t)$. This approach consists in decomposing the $y(t)$ into a truncated Fourier series:

$$y(t) = Y_0 + \sum_{h=1}^{H} \left|Y_{c,h}\right| \cos(h\omega_0 t) + \sum_{h=1}^{H} \left|Y_{s,h}\right| \sin(h\omega_0 t) \tag{8}$$

and substitution of the series into (2). The resultant system of algebraic equations enables to determine the amplitudes of all the harmonics, i.e. Y_0, $Y_{c,h}$ and $Y_{s,h}$, for h = 1,…, H, and thus the periodic solution $y(t)$ expressed by (8).

3. Example I

Let us find the solution of (2) for n = 2, m = 1 and the arc current waveform $i(t)$ based on the exemplary arc current harmonic characteristic limited to the 2nd and 3rd higher harmonics – Fig. 1 (the number of harmonics is consistent with the one used later for the HBM). The percent of fundamental for these harmonics is equal to 8% and 7% (Gomez et al., 2010), respectively. The fundamental frequency is 50 Hz and its amplitude 70 kA. It must be stressed that it is only a simplification of an actual arc current characteristic which is very often used if the analysis is restricted to the deterministic case.

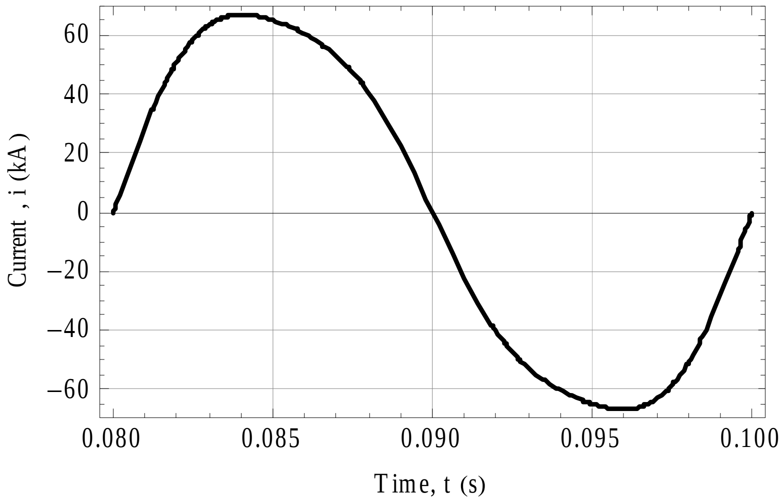

Fig. 1. Exemplary deterministic component of the arc current waveform

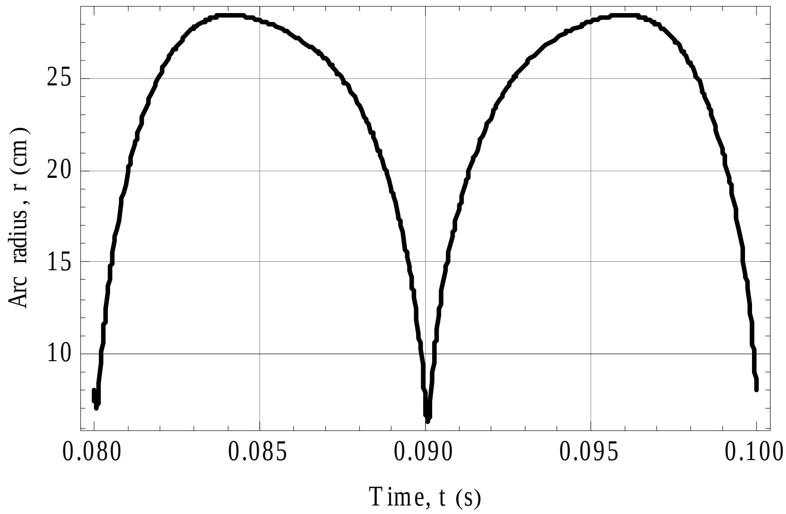

Fig. 2. Arc radius waveform for $n = 2$ and $m = 1$

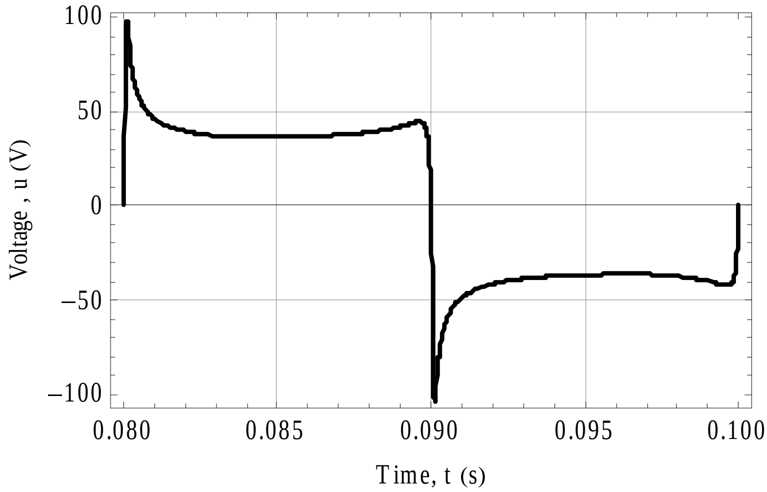

Fig. 3. Arc voltage waveform for $n = 2$ and $m = 1$

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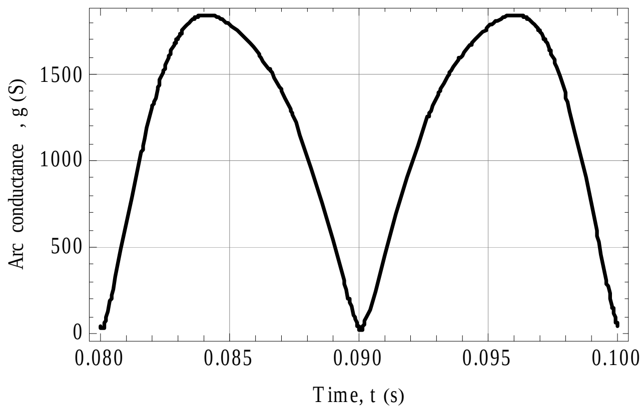

Fig. 4. Arc conductance waveform for n = 2 and m = 1

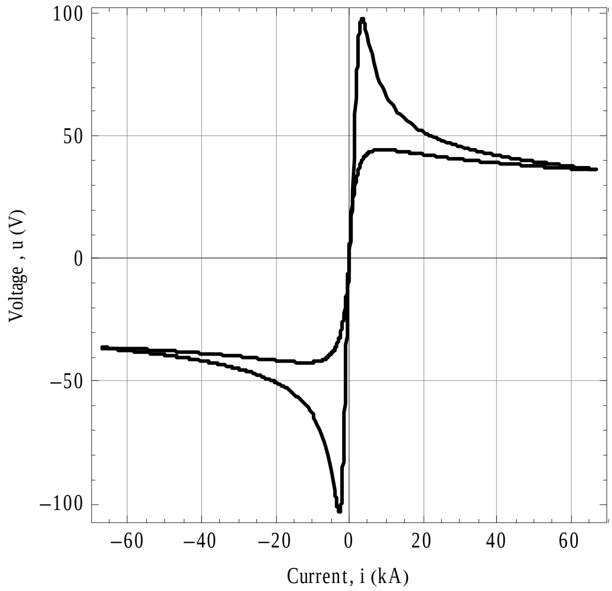

Fig. 5. V-I characteristic of the arc model for n = 2 and m = 1

Let us assume that the constants in equation (1) take the following values: k_1 = 3000, k_2 = 1 and k_3 = 12.5 (Ozgun and Abur, 1999). The solution of (2) enables determination of the arc radius (5) shown in Fig. 2, the arc voltage (6) shown in Fig. 3, the arc conductance (7) shown in Fig. 4 and finally the V-I characteristic shown in Fig. 5.

The V-I characteristics obtained on the base of the closed form solution for n = 2 and m = 0, 1 or 2 are consistent with the numerical solutions and the measured characteristics (Gomez et al., 2010; Ozgun and Abur, 1999; Acha et al., 1990).

4. Quadratic Polynomial Form

The application of the HBM becomes straightforward if a nonlinear equation is transformed to a quadratic form (Cochelina and Vergez, 2009):

$$\mathbf{m}(\dot{\mathbf{z}}) = \mathbf{c}(t) + \mathbf{l}(\mathbf{z}) + \mathbf{q}(\mathbf{z},\mathbf{z}), \tag{9}$$

where:

$\mathbf{m}(\cdot)$, $\mathbf{l}(\cdot)$ – linear vector operators,

$\mathbf{q}(\cdot,\cdot)$ – a quadratic vector operator,

$\mathbf{c}(t)$ – a forcing and constant term vector.

Grabowski D.: *Quadratic polynomial form of electric arc furnace equation*. COMPEL - The International Journal for Computation and Mathematics in Electrical and Electronic Engineering, vol. 33, no. 6, 2014, pp. 2030-2042.

The system of equations (9) can include differential (the dot stands for the derivative with respect to time) and algebraic equations. The vector **z** contains the original unknown variable *y* as well as variables introduced to obtain the quadratic form.

The quadratic polynomial form of (2) for all the values of *k* has been determined and presented below.

If $k = 1/2$ (i.e. $n = 0$, $m = 0$), then:

$$\begin{aligned} \dot{y} &= f(t) + \alpha w + 0 \\ \underbrace{0}_{\mathbf{m}(\dot{\mathbf{z}})} &= \underbrace{0}_{\mathbf{c}(t)} + \underbrace{y}_{\mathbf{l}(\mathbf{z})} - \underbrace{w^2}_{\mathbf{q}(\mathbf{z},\mathbf{z})} \end{aligned} \tag{10}$$

where $\mathbf{z} = [\,y\ w]^{\mathrm{T}}$.

If $k = 3/5$ (i.e. $n = 0$, $m = 1$), then:

$$\begin{aligned} \dot{y} &= f(t) + \alpha w + 0 \\ 0 &= 0 + v - y^2 \\ 0 &= 0 + x - w^2 \\ 0 &= 0 + p - x^2 \\ \underbrace{0}_{\mathbf{m}(\dot{\mathbf{z}})} &= \underbrace{0}_{\mathbf{c}(t)} + \underbrace{0}_{\mathbf{l}(\mathbf{z})} + \underbrace{vy - pw}_{\mathbf{q}(\mathbf{z},\mathbf{z})} \end{aligned} \tag{11}$$

where $\mathbf{z} = [\,y\ v\ x\ p\ w]^{\mathrm{T}}$.

If $k = 2/3$ (i.e. $n = 0$, $m = 2$), then:

$$\begin{aligned} \dot{y} &= f(t) + \alpha w + 0 \\ 0 &= 0 + x - w^2 \\ \underbrace{0}_{\mathbf{m}(\dot{\mathbf{z}})} &= \underbrace{0}_{\mathbf{c}(t)} + \underbrace{0}_{\mathbf{l}(\mathbf{z})} + \underbrace{y^2 - wx}_{\mathbf{q}(\mathbf{z},\mathbf{z})} \end{aligned} \tag{12}$$

where $\mathbf{z} = [\,y\ x\ w]^{\mathrm{T}}$.

If $k = 3/4$ (i.e. $n = 1$, $m = 0$), then:

$$\begin{aligned} \dot{y} &= f(t) + \alpha w + 0 \\ 0 &= 0 + v - y^2 \\ 0 &= 0 + x - w^2 \\ \underbrace{0}_{\mathbf{m}(\dot{\mathbf{z}})} &= \underbrace{0}_{\mathbf{c}(t)} + \underbrace{0}_{\mathbf{l}(\mathbf{z})} + \underbrace{vy - x^2}_{\mathbf{q}(\mathbf{z},\mathbf{z})} \end{aligned} \tag{13}$$

where $\mathbf{z} = [\,y\ v\ x\ w]^{\mathrm{T}}$.
If $k = 4/5$ (i.e. $n = 1$, $m = 1$), then:

Grabowski D.: *Quadratic polynomial form of electric arc furnace equation*. COMPEL - The International Journal for Computation and Mathematics in Electrical and Electronic Engineering, vol. 33, no. 6, 2014, pp. 2030-2042.

$$\begin{array}{l} \dot{y} = f(t) + \alpha w + 0 \\ 0 = 0 + v - y^2 \\ 0 = 0 + x - w^2 \\ 0 = 0 + p - x^2 \\ \underbrace{0}_{\mathbf{m}(\dot{\mathbf{z}})} = \underbrace{0}_{\mathbf{c}(t)} + \underbrace{0}_{\mathbf{l}(\mathbf{z})} + \underbrace{v^2 - pw}_{\mathbf{q}(\mathbf{z},\mathbf{z})} \end{array} \tag{14}$$

where $\mathbf{z} = [\,y\ v\ x\ p\ w]^{\mathrm{T}}$.

If $k = 5/6$ (i.e. $n = 1$, $m = 2$), then:

$$\begin{array}{l} \dot{y} = f(t) + \alpha w + 0 \\ 0 = 0 + v - y^2 \\ 0 = 0 + s - v^2 \\ 0 = 0 + x - w^2 \\ 0 = 0 + p - x^2 \\ \underbrace{0}_{\mathbf{m}(\dot{\mathbf{z}})} = \underbrace{0}_{\mathbf{c}(t)} + \underbrace{0}_{\mathbf{l}(\mathbf{z})} + \underbrace{ys - xp}_{\mathbf{q}(\mathbf{z},\mathbf{z})} \end{array} \tag{15}$$

where $\mathbf{z} = [\,y\ v\ s\ x\ p\ w]^{\mathrm{T}}$.

The results of application of the HBM to (9) obtained for the nonlinear differential equation describing the arc phenomenon have been presented below.

5. Periodic Solution of the Arc Equation

The HBM can be applied to all equations in the quadratic polynomial form obtained in the previous chapter. However, the following considerations have been limited to an exemplary case for which $k = 1/2$. For the other cases similar analysis can be carried out.

If $k = 1/2$, then the arc is described by (10) which can be written in a matrix form:

$$\begin{bmatrix} \dot{y} \\ 0 \end{bmatrix} = \begin{bmatrix} f(t) \\ 0 \end{bmatrix} + \begin{bmatrix} \alpha w \\ y \end{bmatrix} + \begin{bmatrix} 0 \\ -w^2 \end{bmatrix}. \tag{16}$$

In this case the vector operators (see (9)) are defined as follows:

$$\mathbf{l}(\mathbf{z}) = \mathbf{\Lambda z} = \begin{bmatrix} 0 & \alpha \\ 1 & 0 \end{bmatrix} \begin{bmatrix} y \\ w \end{bmatrix}, \tag{17}$$

$$\mathbf{m}(\dot{\mathbf{z}}) = \mathbf{\Gamma} \dot{\mathbf{z}} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{y} \\ \dot{w} \end{bmatrix}, \tag{18}$$

$$\mathbf{c} = \begin{bmatrix} f(t) \\ 0 \end{bmatrix}, \tag{19}$$

Grabowski D.: *Quadratic polynomial form of electric arc furnace equation*. COMPEL - The International Journal for Computation and Mathematics in Electrical and Electronic Engineering, vol. 33, no. 6, 2014, pp. 2030-2042.

$$\mathbf{q}(\mathbf{z},\mathbf{z}) = \begin{bmatrix} q_1(\mathbf{z},\mathbf{z}) \\ q_2(\mathbf{z},\mathbf{z}) \end{bmatrix}, \tag{20}$$

where:

$$q_1(\mathbf{z},\mathbf{z}) = \mathbf{z}^T \mathbf{\Theta}_1 \mathbf{z} = \begin{bmatrix} y & w \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} y \\ w \end{bmatrix}, \tag{21}$$

$$q_2(\mathbf{z},\mathbf{z}) = \mathbf{z}^T \mathbf{\Theta}_2 \mathbf{z} = \begin{bmatrix} y & w \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} y \\ w \end{bmatrix}. \tag{22}$$

The forcing term $f(t)$, which is included in the vector $\mathbf{c}$, as well as the unknown variables can be expressed with the help of the Fourier series:

$$f(t) = F_0 + \sum_{h=1}^{H} F_{c,h} \cos(h\omega_0 t) + \sum_{h=1}^{H} F_{s,h} \sin(h\omega_0 t), \tag{23}$$

$$\mathbf{z}(t) = \mathbf{Z}_0 + \sum_{h=1}^{H} \mathbf{Z}_{c,h} \cos(h\omega_0 t) + \sum_{h=1}^{H} \mathbf{Z}_{s,h} \sin(h\omega_0 t), \tag{24}$$

where:

$$\mathbf{z}(t) = \begin{bmatrix} y(t) \\ w(t) \end{bmatrix}, \tag{25}$$

$$\mathbf{Z}_0 = \begin{bmatrix} Y_0 \\ W_0 \end{bmatrix}, \quad \mathbf{Z}_{c,h} = \begin{bmatrix} Y_{c,h} \\ W_{c,h} \end{bmatrix}, \quad \mathbf{Z}_{s,h} = \begin{bmatrix} Y_{s,h} \\ W_{s,h} \end{bmatrix}. \tag{26}$$

In order to find a periodic solution of the arc equation the Fourier coefficients (26) have to be determined. So a new vector of unknown variables can be introduced:

$$\mathbf{U} = \begin{bmatrix} \mathbf{Z}_0 \\ \mathbf{Z}_{c,1} \\ \mathbf{Z}_{s,1} \\ \vdots \\ \mathbf{Z}_{c,H} \\ \mathbf{Z}_{s,H} \end{bmatrix} = \begin{bmatrix} Y_0 \\ W_0 \\ Y_{c,1} \\ W_{c,1} \\ Y_{s,1} \\ W_{s,1} \\ \vdots \\ Y_{c,H} \\ W_{c,H} \\ Y_{s,H} \\ W_{s,H} \end{bmatrix}. \tag{27}$$

Grabowski D.: *Quadratic polynomial form of electric arc furnace equation*. COMPEL - The International Journal for Computation and Mathematics in Electrical and Electronic Engineering, vol. 33, no. 6, 2014, pp. 2030-2042.

Application of the Fourier series representation of the forcing term and the unknown auxiliary functions allows to convert the quadratic form (9) defined for the unknown vector **z** into a new one defined for the unknown vector **U**:

$$\omega_0 \mathbf{M}(\mathbf{U}) = \mathbf{C} + \mathbf{L}(\mathbf{U}) + \mathbf{Q}(\mathbf{U}, \mathbf{U}), \tag{28}$$

where:

$\mathbf{M}(\cdot)$, $\mathbf{L}(\cdot)$ – linear vector operators,
$\mathbf{Q}(\cdot,\cdot)$ – a quadratic vector operator,
C – a forcing and constant term vector.

The system of equations expressed by (28) implements the harmonic balance method. The operators **L** and **M** are defined on the base of the operators **l** (17) and **m** (18), the vector **C** is derived from the vector **c**:

$$\mathbf{L}(\mathbf{U}) = \begin{bmatrix} \mathbf{l}(\mathbf{Z}_0) \\ \mathbf{l}(\mathbf{Z}_{c,1}) \\ \mathbf{l}(\mathbf{Z}_{s,1}) \\ \mathbf{l}(\mathbf{Z}_{c,2}) \\ \mathbf{l}(\mathbf{Z}_{s,2}) \\ \vdots \\ \mathbf{l}(\mathbf{Z}_{c,H}) \\ \mathbf{l}(\mathbf{Z}_{s,H}) \end{bmatrix} = \begin{bmatrix} \alpha W_0 \\ Y_0 \\ \alpha W_{c,1} \\ Y_{c,1} \\ \alpha W_{s,1} \\ Y_{s,1} \\ \vdots \\ \alpha W_{c,H} \\ Y_{c,H} \\ \alpha W_{s,H} \\ Y_{s,H} \end{bmatrix}, \tag{29}$$

$$\mathbf{M}(\mathbf{U}) = \begin{bmatrix} \mathbf{0} \\ \mathbf{m}(\mathbf{Z}_{s,1}) \\ -\mathbf{m}(\mathbf{Z}_{c,1}) \\ 2\mathbf{m}(\mathbf{Z}_{s,2}) \\ -2\mathbf{m}(\mathbf{Z}_{c,2}) \\ \vdots \\ H\,\mathbf{m}(\mathbf{Z}_{s,H}) \\ -H\,\mathbf{m}(\mathbf{Z}_{c,H}) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ Y_{s,1} \\ 0 \\ -Y_{c,1} \\ 0 \\ \vdots \\ H\,Y_{s,H} \\ 0 \\ -H\,Y_{c,H} \\ 0 \end{bmatrix}, \tag{30}$$

Grabowski D.: *Quadratic polynomial form of electric arc furnace equation*. COMPEL - The International Journal for Computation and Mathematics in Electrical and Electronic Engineering, vol. 33, no. 6, 2014, pp. 2030-2042.

$$\mathbf{C}=\begin{bmatrix}\mathbf{C}_0\\ \mathbf{C}_1\\ \mathbf{C}_2\\ \vdots\\ \mathbf{C}_H\end{bmatrix}=\begin{bmatrix}F_0\\ 0\\ F_{c,1}\\ 0\\ F_{s,1}\\ 0\\ \vdots\\ F_{c,H}\\ 0\\ F_{s,H}\\ 0\end{bmatrix}. \tag{31}$$

In order to define the operator **Q** it is convenient to express q_1 (21) and q_2 (22) by means of the Fourier series:

$$q_1(\mathbf{z},\mathbf{z})=q_1(t)=Q_{10}+\sum_{h=1}^{\infty}\left(Q_{1c,h}\cos(h\omega_0 t)+Q_{1s,h}\sin(h\omega_0 t)\right) \tag{32}$$

$$q_2(\mathbf{z},\mathbf{z})=q_2(t)=Q_{20}+\sum_{h=1}^{\infty}\left(Q_{2c,h}\cos(h\omega_0 t)+Q_{2s,h}\sin(h\omega_0 t)\right) \tag{33}$$

Finally, the operator **Q** is given by:

$$\mathbf{Q}(\mathbf{U},\mathbf{U})=\begin{bmatrix}Q_{10}\\ Q_{20}\\ Q_{1c,1}\\ Q_{2c,1}\\ Q_{1s,1}\\ Q_{2s,1}\\ \vdots\\ Q_{1c,H}\\ Q_{2c,H}\\ Q_{1s,H}\\ Q_{2s,H}\end{bmatrix}=\begin{bmatrix}0\\ Q_{20}\\ 0\\ Q_{2c,1}\\ 0\\ Q_{2s,1}\\ \vdots\\ 0\\ Q_{2c,H}\\ 0\\ Q_{2s,H}\end{bmatrix}, \tag{34}$$

where:

$$Q_{20}=-W_0^2-\frac{1}{2}\sum_{h=1}^{H}W_{c,h}^2-\frac{1}{2}\sum_{h=1}^{H}W_{s,h}^2\ , \tag{35}$$

$$Q_{2c,2h-1}=-2W_0W_{c,2h-1}-\sum_{\substack{j=1\\ h\neq 1}}^{2h-3}\left(W_{c,j}W_{c,2h-1-j}-W_{s,j}W_{s,2h-1-j}\right)-\sum_{\substack{j=2h+1\\ 2h-1\neq H}}^{H}\left(W_{c,j}W_{c,j-2h+1}+W_{s,j}W_{s,j-2h+1}\right), \tag{36}$$

$$Q_{2c,2h} = -2W_0 W_{c,2h} - \sum_{\substack{j=1 \\ h\neq 1}}^{2h-2} \left(W_{c,j} W_{c,2h-j} - W_{s,j} W_{s,2h-j} \right) - \sum_{\substack{j=2h+2 \\ 2h\neq H}}^{H} \left(W_{c,j} W_{c,j-2h} + W_{s,j} W_{s,j-2h} \right) - \frac{1}{2} W_{c,h}^2 + \frac{1}{2} W_{s,h}^2, \tag{37}$$

$$Q_{2s,h} = -2W_0 W_{s,h} - \sum_{\substack{j=1 \\ h\neq 1}}^{h-1} \left(W_{c,j} W_{s,h-j} + W_{s,j} W_{c,h-j} \right) - \sum_{\substack{j=h+1 \\ h\neq H}}^{H} \left(W_{c,j-h} W_{s,j} - W_{s,j-h} W_{c,j} \right). \tag{38}$$

The vector (34) has been obtained assuming that the Fourier series (32) and (33) are truncated and include only the H harmonics. Substituting (29), (30), (31) and (34) into (28) and solving it allows to determine the periodic solution $y(t)$ of the arc equation (2) for $k = 1/2$.

6. Example II

Let us assume that the arc current harmonic values as well as the constants in (1) take the same values as in Example I. Solving (28) enables among others determination of the harmonics for the unknown function $y(t)$ (see (27)), which allows to calculate the arc radius and voltage on the base of (5) and (6), respectively. As in the previous chapter, the case of $k = 1/2$ (i.e. $n = 0$ and $m = 0$) has been considered in the example. The system of equations defined by (28) after canceling any harmonics higher than the assumed maximum index $H = 3$ has been solved. The harmonic content of the periodic solution $y(t)$ has been presented in Tab. I (for solutions from #1 to #4 the RMS value of the first harmonic has been used as the reference value, for solution #5 it has been the RMS value of the second harmonic, because the first one is not present in the spectrum).

TABLE I
HBM RESULTS - HARMONIC PERCENTAGE

Solution	Harmonic					
	h=1		***h=2***		***h=3***	
	$Y_{C,1}$	$Y_{S,1}$	$Y_{C,2}$	$Y_{S,2}$	$Y_{C,3}$	$Y_{S,3}$
#1	48.6%	87.4%	26.4%	20.7%	53.9%	2.6%
#2	55.6%	83.1%	80.6%	45.3%	52.9%	10.6%
#3	59.4%	80.4%	29.8%	2.1%	19.4%	6.9%
#4	62.1%	78.4%	43.7%	5.9%	18.6%	8.8%
#5	0.0%	0.0%	0.0%	100%	0.0%	0.0%

If the analysis is limited to harmonics of order 3 or less, then there are five real solutions which have been found using the HBM. The more deep discussion of results will be possible after finishing a dedicated software program which enables finding periodic solutions for $k \in [0.5;1]$ and for varying parameter α.

7. Conclusions

The nonlinear differential equation used to model the electric arc furnace has been transformed to the quadratic polynomial form. Since the nonlinearities are quadratic, the HBM can be easily applied to the transformed equation, even with a large number of harmonics. In order to follow periodic solutions of the arc equation when its parameters are varied a continuation method

Grabowski D.: *Quadratic polynomial form of electric arc furnace equation*. COMPEL - The International Journal for Computation and Mathematics in Electrical and Electronic Engineering, vol. 33, no. 6, 2014, pp. 2030-2042.

can be applied in the next step. The computational results obtained so far agree with existing numerical solutions and measurements.

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