Deterministic model of electric arc furnace - a closed form solution

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Abstract

Purpose – Electric arc furnaces are usually modelled using combined models which divide the phenomenon taking place in real objects into a deterministic and a stochastic or chaotic parts. The former is expressed by a nonlinear differential equations. The paper goal was to obtain a closed form of the solution to one of the most popular nonlinear differential equations used for the AC electric arc modelling.
Design/methodology/approach – The solution has been obtained in the time domain by a sequence of transformations of the original nonlinear equation which lead to a linear equation, for which a closed form solution is known.
Findings – A set of parameters for which the solution to the nonlinear differential equation describing electric arc can be obtained in a closed form.
Research limitations/implications – There are still some parameter values for which the solution can be found only numerically. Moreover, due to the nature of the phenomena occurring in electric arc furnaces in order to build a complete model of the arc the deterministic model must be extended using for example stochastic approach.
Practical implications – The obtained results enable determination of exact waveforms of the arc voltage or radius without application of numerical algorithms for ODE solving. The arc model can be used to evaluate the impact of arc furnaces on power quality during the planning stage of new plants. The proposed approach facilitates calculation of the arc characteristic.
Originality/value – The importance of having a closed form of the solution instead of the numerical ones comes from new possible ways of extension of the arc model in order to cover the time-varying nature of the arc waveforms. So far the equation has been solved only using numerical algorithms.
Keywords – electric arc furnace, nonlinear differential equations, closed form solution, nonlinear loads
Paper type – Research paper

1. Introduction

Electric arc furnaces are commonly used in steel industry. Unfortunately, nonlinear characteristic of the arc furnace and its stochastic behaviour bring about many problems, e.g. voltage flicker and waveform distortions (Gomez *et al.*, 2010). This is the reason for many studies which have been carried out last years in order to get better understanding of the phenomenon and to find out an advanced model reflecting the complex nature of the electric arc (Ramirez, 2000). It is especially hard task due to the stochastic character of processes taking place in the furnace during melting – the arc extinguishes and starts again in a random way (Moller *et al.*, 1980). Thus, the most popular approach consists in two-step modeling – first the simplified model taking into account only a deterministic component of the solution is analyzed and then the time-varying character of the arc is included by adding a chaotic, a stochastic or a modulated component to the deterministic solution (Alvesa *et al.*, 2010; Golkar and Meschi, 2008; Gomez *et al.*, 2010; Ozgun and Abur, 1999; Walczak and Piwowar, 2010). It should be stressed that the analysis is carried out using numerical algorithms for solving nonlinear differential equations.

This paper presents a closed form solution obtained for a nonlinear differential equation used commonly for the deterministic AC arc modeling (Acha *et al.*, 1990). It extends results presented in (Grabowski and Walczak, 2011). The closed form solution makes the analysis faster and more

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reliable. It also enables to obtain a realistic arc model by treating some parameters of the arc as stochastic variables.

2. Deterministic arc furnace model

2.1. The arc equation

The paper shows results obtained in the field of deterministic V-I arc characteristic modelling by means of a nonlinear differential equation that comes from the instantaneous power balance equation (Acha *et al.*, 1990; Gomez *et al.*, 2010; Ozgun and Abur, 1999). The other approaches consist in approximation based on a step function (Bellido and Gomez, 1997), a piece-wise linear or an exponential approximation (Golkar and Meschi, 2008) and a neural network black-box model (Chang *et al.*, 2010).

The power balance equation of the arc furnace which is the starting point of the method results in the following nonlinear differential equation describing a single-phase electric arc (Acha *et al.*, 1990):

$$k_1 r^n(t) + k_2 r(t)\frac{dr(t)}{dt} = \frac{k_3}{r^{m+2}(t)} i^2(t) \quad (1)$$

where $r(t)$ - the arc radius, $i(t)$ - the arc current, k_1, k_2, k_3 - the model coefficients determined experimentally, n and m – the equation parameters.

In this equation the arc radius is regarded as an unknown variable and the arc current $i(t)$ as input data. Subsequently, the arc voltage $u(t)$ can be determined (Acha *et al.*, 1990):

$$u(t) = \frac{k_3}{r^{m+2}(t)} i(t) \quad (2)$$

So the arc conductance $g(t)$ as a function of the arc radius $r(t)$ can be expressed by:

$$g(t) = \frac{r^{m+2}(t)}{k_3} \quad (3)$$

The last equation can be transformed to obtain the inverse relation, i.e. the arc radius expressed as a function of the arc conductance:

$$r(t) = \left(k_3 g(t)\right)^{\frac{1}{m+2}} \quad (4)$$

Equation (1) can be also extended to cover the case of a three-phase electric arc (Gomez *et al.*, 2010).

The values of the exponents n and m are limited to 0, 1 or 2 and they reflect different working conditions depending on the arc furnace cycle (Bellido and Gomez, 1997). The most popular coefficient values are $n = 2$ and $m = 0$ (Acha *et al.*, 1990; Gomez *et al.*, 2010; Ozgun and Abur, 1999). Such coefficient set implies that one is modeling the melting stage of the arc, which is the operation stage corresponding to the worst condition regarding both flicker and harmonic generation. Other operation stages are, usually, not considered when the simulation is aimed at harmonic distortion or flicker calculations.

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The closed form solution to equation (1) for this special case (n = 2 and m = 0) has been derived in (Grabowski and Walczak, 2011). The question is: does a closed form solution exist for the other values of n and m?

2.2. Closed form solution to the arc equation

Multiplying the arc equation (1) by $r^{m+2}(t)$ gives:

$$k_1 r^{n+m+2}(t) + k_2 r^{m+3}(t)\frac{dr(t)}{dt} = k_3 i^2(t) \tag{5}$$

Using the following substitution:

$$y(t) = r^{m+4}(t), \quad \frac{dy(t)}{dt} = (m+4)\, r^{m+3}(t)\frac{dr(t)}{dt} \tag{6}$$

results in a new form of the nonlinear arc equation:

$$\frac{dy(t)}{dt} = \alpha\, y^k(t) + f(t) \tag{7}$$

where:

$$f(t) = \frac{(m+4)k_3}{k_2} i^2(t) \tag{8}$$

$$\alpha = -\frac{(m+4)k_1}{k_2} = const \tag{9}$$

$$k = \frac{n+m+2}{m+4} \tag{10}$$

Taking into account that the coefficients n and m can take values 0, 1 or 2, all possible cases of the exponent k (10) and equation (7) have been put together in table I.

So the arc equation for n = 2 and m = 0, 1 or 2 can be brought to a linear equation – the conditions of existence and uniqueness of its solutions are fulfilled assuming that the function $f(t)$ is bounded. Its solution is expressed by (Zwillinger, 1997):

$$y(t) = (m+4)\frac{k_3}{k_2} e^{-\beta t}\int_0^t i^2(\tau) e^{\beta\tau} d\tau, \quad \beta = (m+4)\frac{k_1}{k_2} \tag{11}$$

So on the base of equation (6):

$$r(t) = y^{\frac{1}{m+4}}(t) \tag{12}$$

and finally the arc voltage and conductance can be determined using equations (2) and (3), respectively.

TABLE I
THE ARC FURNACE EQUATIONS

n	***m***	***k***	**d*y*/d*t* - equation (7)**
0	0	1/2	$\alpha y^{1/2} + f(t)$
	1	3/5	$\alpha y^{3/5} + f(t)$
	2	2/3	$\alpha y^{2/3} + f(t)$
1	0	3/4	$\alpha y^{3/4} + f(t)$
	1	4/5	$\alpha y^{4/5} + f(t)$
	2	5/6	$\alpha y^{5/6} + f(t)$
2	0	1	$\alpha y + f(t)$
	1	1	$\alpha y + f(t)$
	2	1	$\alpha y + f(t)$

For n = 0 or 1 the arc equation must be solved using other methods because a closed form probably does not exist (Polyanin and Zaitsev, 2003).

2.3. Example

The arc current harmonic characteristic can be found in many publications (Brociek and Wilanowicz, 2011; Chang *et al.*, 2008; Ghoudjehbaklou and Kargar, 2002; Salor *et al.*, 2010). The arc current waveform $i(t)$ (Fig. 1) used for simulation has been based on the exemplary arc current harmonic characteristic with relatively low distortion level - the only nonzero harmonics are 3rd, 5th and 7th. The percent of fundamental for these harmonics is equal to 5%, 4.5% and 1%, respectively. Of course, it is only a deterministic simplification of an actual arc furnace current. The comparison of results obtained for different percentages of the harmonics have been shown in (Grabowski and Walczak, 2011).

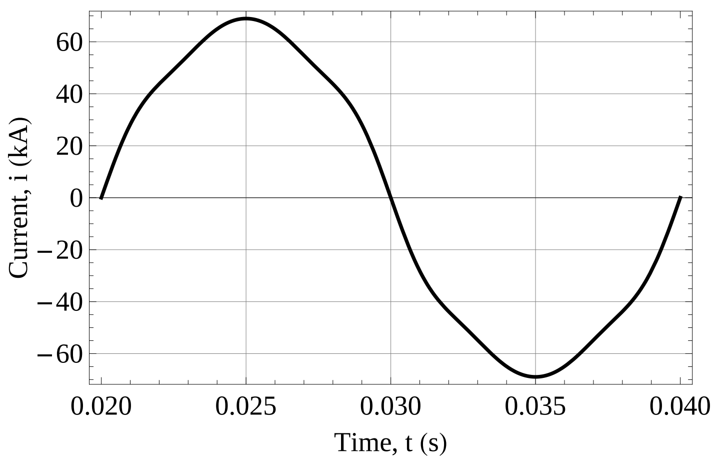

Fig. 1. Arc current waveform

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Let us assume that the constants in equation (1) take the following values: k_1 = 3000, k_2 = 1 and k_3 = 12.5 (Ozgun and Abur, 1999). Solution (11) enables determination of the arc radius (12) shown in Fig. 2, the arc voltage (2) shown in Fig. 3, the arc conductance (3) shown in Fig. 4 and finally the V-I characteristic shown in Fig. 5 for n = 2 and m = 0, 1, 2.

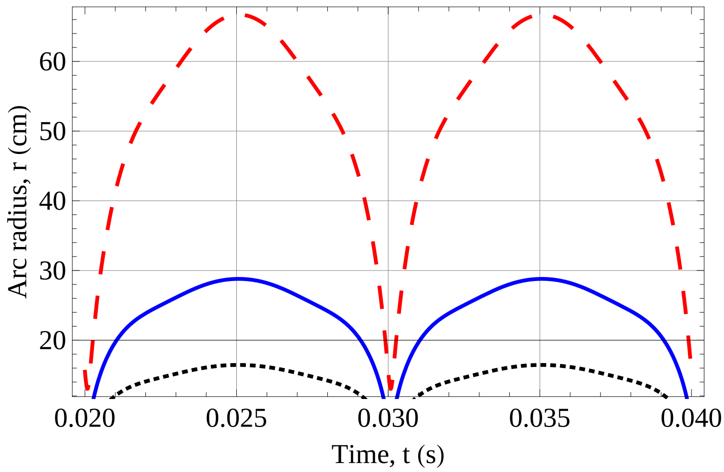

Fig. 2. Arc radius waveform for n = 2 and m = 0 (dashed line), m = 1 (solid line) and m = 2 (dotted line)

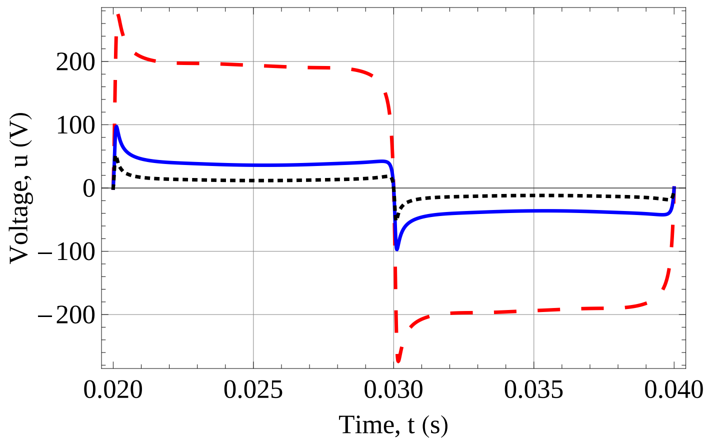

Fig. 3. Arc voltage waveform for n = 2 and m = 0 (dashed line), m = 1 (solid line) and m = 2 (dotted line)

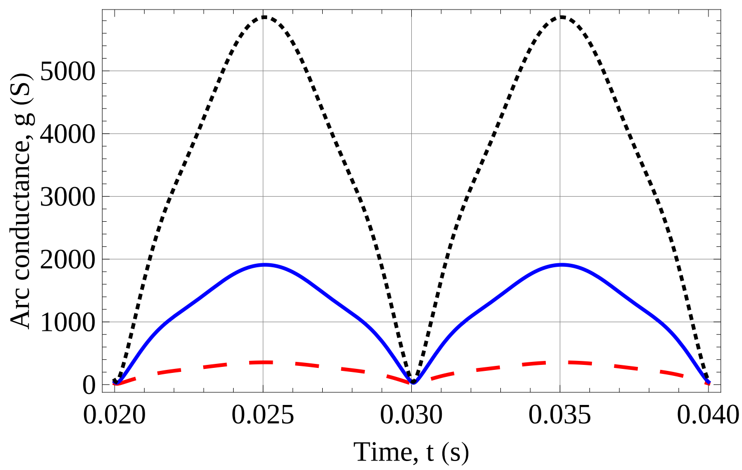

Fig. 4. Arc conductance waveform for n = 2 and m = 0 (dashed line), m = 1 (solid line) and m = 2 (dotted line)

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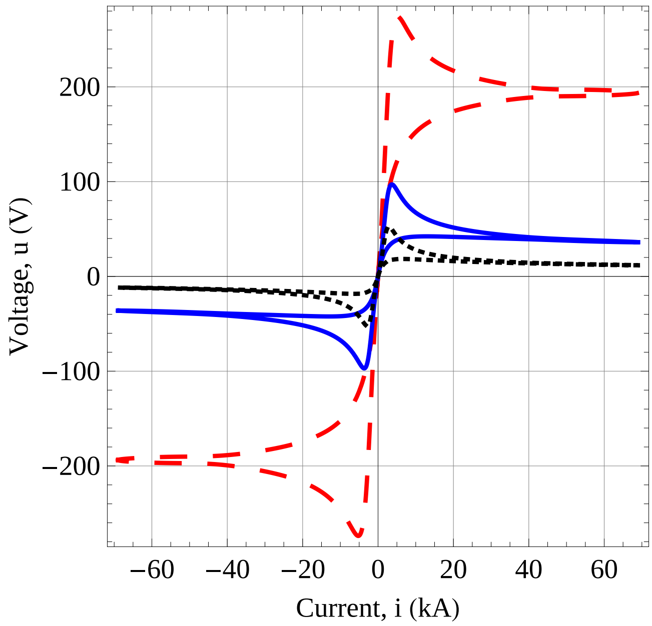

Fig. 5. V-I characteristic of the arc model for n = 2 and m = 0 (dashed line), m = 1 (solid line) and m = 2 (dotted line)

In order to compare the closed form solution defined by equation (11) and the numerical one the V-I characteristics for $n = 2$ and $m = 0$ obtained using both approaches have been shown in Fig. 6.

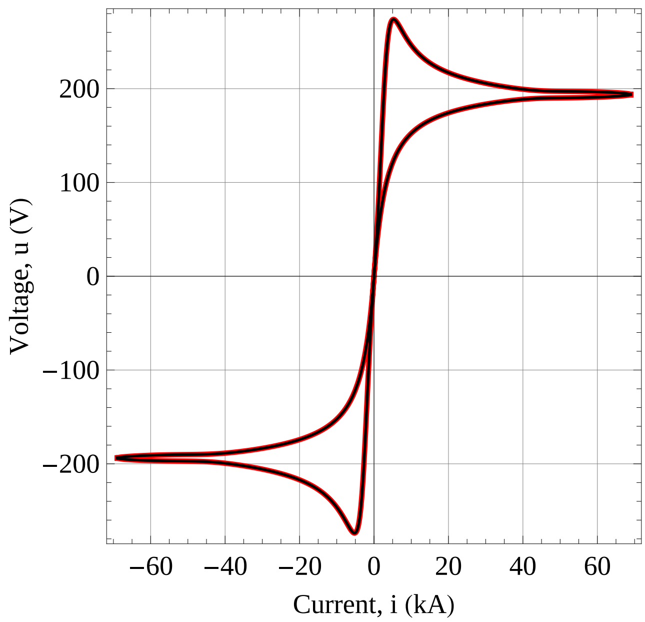

Fig. 6. V-I characteristic of the arc model for n = 2 and m = 0 – the closed form solution (black line) and the numerical solution (red line)

The numerical solution has been obtained using *Mathematica* software package. The *Mathematica* function NDSolve belongs to the most advanced general numerical differential equation solvers. It can handle a very wide range of ordinary differential equations. NDSolve has several methods built in including explicit Runge-Kutta methods. The methods are reentrant and

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hierarchical, what means that one method can call another. Moreover, there is a mechanism which provides automatic step-size selection and method-order selection (Hunt et al., 2009).

The V-I characteristics for $n = 2$ and $m = 0$ (Fig. 6) obtained on the base of the closed form solution defined by equation (11) is consistent not only with the numerical solution but also with the measured characteristics reported by other authors (Acha *et al.*, 1990; Ghoudjehbaklou and Kargar, 2002; Gomez *et al.*, 2010; Ozgun and Abur, 1999).

2.4. Numerical solution to the arc equation

Equation (7) in the case of non-integer values of k (see Table I) can be solved numerically using one of the engineering software packages. This section contains a proposition of a method which also leads to such solution.

The discrete equivalent of equation (7) for sampling period T is given by:

$$y[n] = y[n-1] + T\alpha\left(y[n]\right)^k + Tf[n] \tag{13}$$

It can be written down as:

$$y[n] - T\alpha\left(y[n]\right)^k = \Psi[n, n-1] \tag{14}$$

where $\Psi[n, n\text{-}1]$ is known and expressed by:

$$\Psi[n, n-1] = y[n-1] + Tf[n] \tag{15}$$

So assuming $y[n] \mathrel{\widehat{=}} y$ and $\Psi[n, n\text{-}1] \mathrel{\widehat{=}} b$ the following algebraic equation must be solved in each step n:

$$y - T\alpha y^k = b \tag{16}$$

Moving linear terms to the left side and nonlinear to the right one gives:

$$y - b = \xi(y) \tag{17}$$

where:

$$\xi(y) = T\alpha y^k \tag{18}$$

Graphical illustration of solutions to equation (17) for all cases of the exponent k, i.e. $k = 1/2$ (blue curve), $k = 3/5$ (brown curve), $k = 2/3$ (black curve), $k = 3/4$ (green curve), $k = 4/5$ (gray curve) and $k = 5/6$ (purple curve) - see Table I, has been denoted by red dots in Fig. 7 for exemplary values of the coefficients $T = 1$, $\alpha = 1$ and $b = 1$. The solutions are determined by intersections of monotonic functions (18) and a linear function defined by the left side of equation (17). Equivalently, they can be easily found as roots of functions obtained by moving the right side of equation (17) to the left side – see Fig. 8**.** Assuming that $b > 0$ the roots are single and should be searched within the interval (b, ∞).

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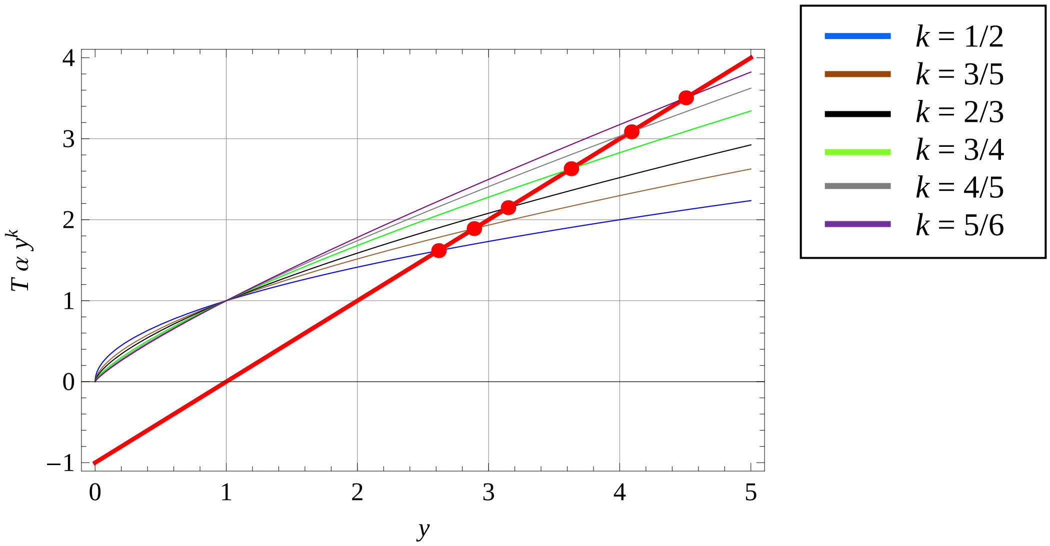

Fig. 7. Solutions to equation (17) as intersections for different values of the exponent k

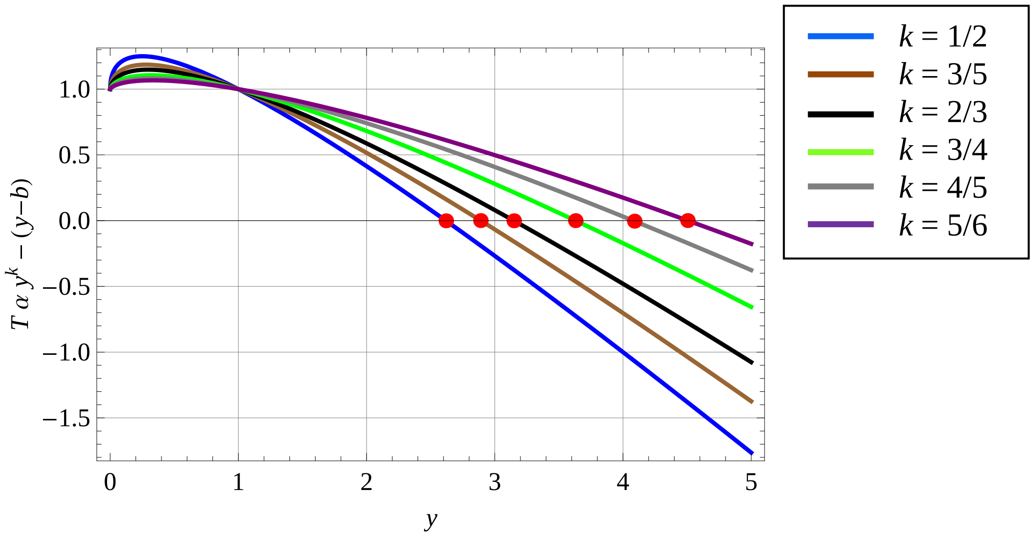

Fig. 8. Solutions to equation (17) as roots of functions for different values of the exponent k

The presented method can be realized by a nonlinear IIR (Infinite Impulse Response) digital filter which implements equation (14).

3. Conclusions

A differential equation describing the deterministic behaviour of an AC electric arc has been considered in the paper. The conditions which must be fulfilled in order to get a closed form solution to this equation have been given. The proposed approach makes calculation of the arc characteristic easier and enables in the future relatively simple extension of the model in order to reflect the arc stochastic nature. The computational results agree with existing numerical solutions and measurements.

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