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Kaluzhnin's representations of Sylow p-subgroups of automorphism groups of p-adic rooted trees

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Abstract. The paper concerns the Sylow p-subgroups of automorphism groups of level homogeneous rooted trees. We recall and summarize the results obtained by L. Kaluzhnin on the structure of sylow p-subgroups of isometry groups of an ultrametric Cantor p-spaces in terms of automorphism groups of rooted trees. Most of the paper should be viewed as a systematic topical survey, however we include some new ideas in last sections.

1. Introduction

The Sylow p-subgroup P_∞ of the automorphism group of a p-adic rooted tree is one of the most popular examples of pro-p groups.This is due to the universality of P_∞ in the classes of all countable residually-p groups and profinite groups of countable weight. The group has natural characterizations in topological terms (as the group of isometries of Cantor metric space), geometrical terms (as a group of automorphisms of a rooted tree) and algebraic terms (as an infinitely iterated wreath product of cyclic groups of order p). It may be also defined as the limit of the inverse spectrum of Sylow p-subgroups in symmetric groups of orders p, p^2, p^2,..., etc. If $p = 2$ the group P_∞ coincides with the whole automorphism group of the binary tree.

It is worth mentioning that all known examples of residually finite groups of Burnside type may be embedded in the Sylow p-subgroup P_∞ of the automorphism group of a p-adic rooted tree, and most of them were

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obtained just as subgroups of P_∞. Therefore P_∞ remains an interesting object of investigations and important for the overall theory.

The group P_∞ has been introduced almost 70 years ago by L. Kaluzhnin in his note [12] as a generalisation of the finite groups P_m, more precisely as an inverse limit of the system of groups P_m, $m \in \mathbb{N}$. The note was a part of a series of papers on Sylow p-subgroups of symmetric groups [8, 9, 10, 11, 13, 12, 14]. In his paper [14] professor Kaluzhnin summarized his results on P_∞. The main of these are:

(i) characterization of P_∞ as a Sylow p-subgroup of isometry group of an ultrametric Cantor p-space, metrization of P_∞ and investigations of its universal properties among so called p_∞-groups;

(ii) introduction of the concept of parallelotopic subgroups and determination of necessary and sufficient conditions for these subgroups to be normal in P_∞;

(iii) characterization of invariant to isometric automorphisms subgroups of P_∞.

The results (i)–(iii) were obtained as generalisations of the respective results in groups P_m. After Kaluzhnin's papers the group structure of P_∞ has not been much investigated. Some results concerning the automorphism group of P_∞ were obtained by P. Lentoudis in [20], who studied isometric automorphisms and gave their complete description. In [1] M. Abert and B. Virag discussed some probabilistic properties of groups acting on rooted trees. In [22] V. Sushchanskyy discussed embeddings of residually finite p-groups into the groups of isometries of the space of p-adic integers. Some facts regarding this group follow from investigations of infinitely iterated wreath products of groups [15], branch groups [2] and some others [6, 7].

In the presented paper we give a systematic overview on the results known on the structure of the group P_∞ and provide a few new facts in this topic. In Section 2 we introduce the notion of a (finite) rooted p-tree and discuss the group of its automorphisms. Section 3 concerns automorphism groups of infinite p-trees and provides a useful representation of elements of such groups. In Section 4 we discuss the standard embedding of residually finite groups as topological groups (the coset representation) into the automorphism groups of some infinite level homogeneous tree. We also deduce and recall the universal properties of the group P_∞. In Section 5 we discuss another representation of elements of P_∞, proposed by L. Kaluzhnin for p-elements of automorphism groups either finite or infinite p-adic rooted trees. Some basic results on this notation and its properties are recalled here after [13]. We note that some generalization of Kaluzhnin's representation is proposed and investigated in [4].

Section 6 is devoted to subgroups in P_∞ of a special type - the so called ideal subgroups. We recall and prove a number of observations on ideal subgroups of P_∞ and use it in Sections 7 and 8 for discussion on the lower central series and isometrically characteristic subgroups of P_∞.

2. Rooted p-trees of finite height and groups of their automorphisms

Let $T(v)$ denote a rooted tree, i.e. a connected graph with no cycles and a designated vertex v called the root. Every two vertices u and w of the tree are connected with a unique path. The length of this path, i.e. the number of edges in the path, is called the distance between u and w. The set of all vertices of the tree $T(v)$ can be partitioned into subsets, called levels, with respect to the distance of the given vertex to the root v. The n-th level, denoted by L_n is defined by the distance n, and $L_0 = \{v\}$. Given a vertex $w \in L_n$ the number of vertices in L_{n+1} which are adjacent to w is called the (branch) index of w. A rooted tree is called level-homogeneous if the indices of all vertices from each level are equal. For brevity, we will call the common index of the vertices in level L_n - the index of level L_n. It is clear that the index level of L_n is equal to $|L_n|/|L_{n-1}|$. A level homogeneous tree can be uniquely characterized (up to isomorphism) by a sequence (either finite or infinite) $\overline{\kappa} = (n_1, n_2, ...,)$ of level indices; we denote this tree by $T_{\overline{\kappa}}$. If there exists N, such that $L_N \neq \emptyset$ and $L_{N+k} = \emptyset$ for all $k > 0$, then N is called the height of the rooted tree.

In this paper we are concerned with the p-trees, which are defined to be level-homogeneous rooted trees such that the indices related to every level of the tree (except the last level) are p-powers increased (p - a prime). Namely, given a sequence $\overline{p} = \{p^{n_i}\}_{i=1}^{N}$ of natural powers of a prime p, we define a rooted p-tree $T_{\overline{p}}(v, N)$ with root v as a level - homogeneous tree in which the i-th level has the index p^{n_i}, $1 < i < N$ and the last level has index 0. One special example of a p-tree is the p-adic rooted tree $T_p(v, N)$, in which all level indices, except those in the first and the last level, are equal to p (i.e. the sequence $\overline{p}$ is constant).

A bijection τ from the set of vertices of T to itself is called an automorphism of the tree T, if it preserves the adjacency of vertices. When discussing automorphisms of a rooted tree, one additionally requires an automorphism to preserve the root. The set of all automorphisms of the tree T with the operation of morphism composition constitutes a group, which we will denote by $\operatorname{Aut} T$. Let us first recall the well known result on the structure of $\operatorname{Aut} T_{\overline{p}}(v, N)$:

Proposition 2.1. *Let $\overline{p} = \{p^{n_i}\}_{i=1}^{N}$ be a sequence of natural powers of prime p. Then*

$$\operatorname{Aut} T_{\overline{p}}(v, N) \cong \mathop{\wr}_{i=1}^{N} S_{p^{n_i}}.$$

In particular, for a p-adic rooted tree $T_p(v, N)$ of height N we have

$$\operatorname{Aut} T_p(v, N) \cong \mathop{\wr}_{i=1}^{N} S_p^{(i)}.$$

The proposition shows the mutual correspondence of groups of automorphisms of a p-adic rooted tree and certain transformation groups. This can be easily seen by studying the action of a p-adic rooted tree automorphism on the last level of the tree. Every such action is a permutation from S_{p^N}, where N is the height of the tree.

In his papers [8, 9, 10, 11, 13], L. Kaluzhnin studied the Sylow p-subgroups of the symmetric group S_{p^n}. His investigations begin with the following results:

Lemma 2.2. *The Sylow p-subgroup $Syl_p(S_{p^N})$ of the symmetric group S_{p^N} is isomorphic to a N-iterated wreath power of cyclic groups C_p of degree p:*

$$Syl_p(S_{p^N}) \cong \mathop{\wr}_{i=1}^{N} C_p^{(i)}.$$

This result may be extended to the automorphism group of any rooted p-tree:

Proposition 2.3. *Let $\overline{p} = \{p^{n_i}\}_{i=1}^{N}$ be a sequence of natural powers of prime p. Then every Sylow p-subgroup $Syl_p(\operatorname{Aut} T_{\overline{p}}(v, N))$ is isomorphic to the M-iterated wreath power of cyclic groups of degree p:*

$$Syl_p(\operatorname{Aut} T_{\overline{p}}(v, N)) \cong \mathop{\wr}_{i=1}^{M} C_p^{(i)},$$

where $M = n_1 + n_2 + ... + n_N$.

Proof. It is enough to see that the iterated wreath power of permutation groups is associative. Then we have:

$$Syl_p(\operatorname{Aut} T_{\overline{p}}(v, N)) \cong \mathop{\wr}_{i=1}^{N} Syl_p(S_{p^{n_i}}) \cong \mathop{\wr}_{i=1}^{N} \mathop{\wr}_{j=1}^{n_i} C_p^{(j)} \cong \mathop{\wr}_{i=1}^{M} C_p^{(i)},$$

where M stands for the sum of all n_i, $i = 1, ..., N$. □

The above proposition shows in particular, that the study of Sylow p-subgroup of a rooted p-tree bears the same class of groups as Sylow p-subgroups of a p-adic rooted tree.

3. Automorphism groups of infinite p-trees

So far we have discussed only finite trees, i.e. trees with finitely many vertices. An infinite tree is said to be locally finite, if the branch index of every vertex in that tree is finite. As particular examples of locally finite trees one may consider the (infinite) rooted p-tree $T_{\overline{p}}(v)$ and an (infinite) p-adic rooted tree $T_p(v)$. These are trees with infinitely many levels (hence no leaves) and infinitely many infinite paths starting at root (rays). The uncountably infinite set $\partial T_{\overline{p}}(v)$ of all rays is called the boundary of the tree. In the set $\partial T_{\overline{p}}(v)$ we introduce a metric Δ in the following way. For two rays $\alpha, \beta \in \partial T$ we set the distance as $\Delta(\alpha, \beta) = \frac{1}{2^k}$, where k is the highest number of the level, such that both α and β contain a common vertex in L_k. If such level does not exist, then the rays coincide and we set $\Delta(\alpha, \beta) = 0$. Then $(\partial T, \Delta)$ is a compact ultrametric space.

Let $\operatorname{Aut} T_{\overline{p}}(v)$ be the group of automorphisms of the infinite p-adic rooted tree and let P_∞ be its Sylow p-subgroup. We have:

Proposition 3.1. *For every infinite sequence $\overline{p} = \{p^{n_i}\}_{i=1}^{\infty}$ of positive powers of a prime p the Sylow p-subgroup of $\operatorname{Aut} T_{\overline{p}}(v)$ is isomorphic to*

$$P_\infty \cong \mathop{\wr}_{i=1}^{\infty} C_p^{(i)}.$$

Remark 3.2. We mention that if $p = 2$ then the Sylow 2-subgroup of $\operatorname{Aut} T_{\overline{2}}(v)$ coincides with $\operatorname{Aut} T_{\overline{2}}(v)$ if and only if rhe sequence $\overline{2}$ is constant, i.e. $\overline{2} = (2, 2, 2, ...)$.

We mention here that P_∞ can be alternatively constructed as an inverse limit of the inverse system (P_n, ψ_n) of Sylow p-subgroups in automorphism groups of finite trees $T_p(v, n)$, with natural projections $\psi_n : P_{n+1} \hookrightarrow P_n$, $n \in \mathbb{N}$ (acting by neglecting the action on the last level of the tree).

We now recall a useful and illustrative notation for elements of $\operatorname{Aut} T_p(v)$ – the portrait of automorphisms of $T_p(v)$. If $f \in \operatorname{Aut} T_p(v)$ is an automorphism, then at every level L_{n+1} of the tree f acts as a permutation with distinct cycles on every set of descendants of vertices in L_n. This is imposed by the requirement on automorphism to preserve the adjacency of vertices. Hence, at the level L_{n+1} one may illustrate the action of f as shown in Fig.1.

Every automorphism $f \in \operatorname{Aut} T_p(v)$ acts in a natural way on the boundary $\partial T_p(v)$. Hence $\operatorname{Aut} T_{\overline{p}}(v)$ acts on the compact ultrametric space $(\partial T, \Delta)$. Therefore the group $\operatorname{Aut} T_{\overline{p}}(v)$ is metrizable in a standard way.

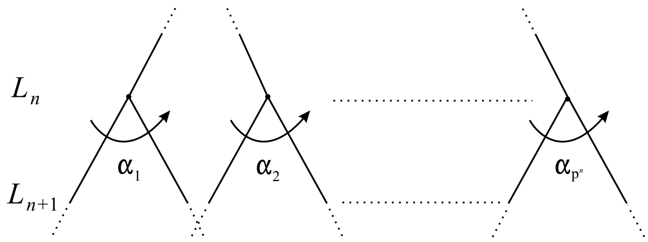


Fig. 1: A part of the portrait of an automorphism $f \in \operatorname{Aut} T_p(v)$.

Namely, for any two automorphisms $f, g \in \operatorname{Aut} T_p(v)$ we put

$$d(f,g) := \max_{\alpha \in \partial T}(\alpha^f, \alpha^g).$$

According to this definition the distance $d(f,g) = 2^{-k}$, where k is the number of the highest level in $T_p(v)$, such that the portraits of automorphisms f and g are the same on the first k levels of $T_p(v)$. If such a number does not exist, i.e. the automorphisms f and g have identical portraits, then we set $d(f,g) = 0$. The topology induced on $\operatorname{Aut} T_p(v)$ by the ultrametric d is a standard profinite topology.

4. Residually finite groups as automorphism groups of trees

Let G be a countably generated residually finite group and let Σ be a series of subgroups of G:

$$G = G_1 \supseteq G_2 \supseteq G_3 \supseteq ..., \tag{1}$$

such that $\bigcap_{i=1}^{\infty} G_i = \{1\}$ and the factor groups G_i/G_{i+1} are finite. We construct the coset rooted tree $T(\Sigma)$ in the following way. The root corresponds to the whole group G, and for $i \geq 1$ the vertices of the i-th level of $T(\Sigma)$ are the right cosets G/G_i. A vertex $G_i x$ of i-th level is connected with a vertex $G_{i+1} y$ of the $i+1$-st level ($i \geq 0$), if and only if $G_{i+1} y \subset G_i x$. The constructed rooted tree is level-homogeneous with level indices equal to $s_i = [G_i : G_{i+1}]$ (see Fig. 2). As the level indices are finite, $T(G)$ is a locally finite tree.

It is well known that the function δ defined on G by the equation

$$\delta(x,y) = 2^{-k} \quad \Leftrightarrow \quad xy^{-1} \in G_k \setminus G_{k+1},$$

for every $x, y \in G$ such that $x \neq y$ and such that $\delta(x,x) = 0$ is a metric on G. It is easy to check that δ is an ultrametric on G. Namely, if

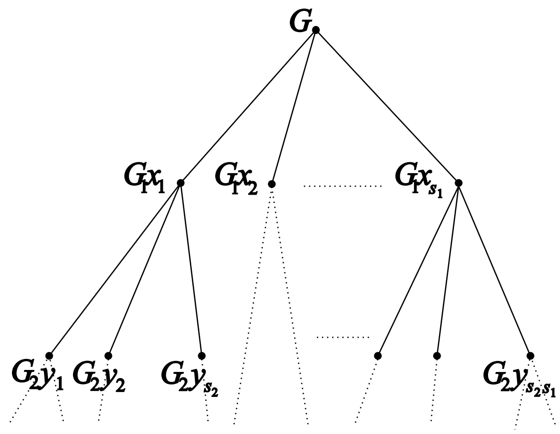


Fig. 2: Construction of a coset tree $T(\Sigma)$.

$x \neq y$ and $\delta(x,y) = 2^{-k}$ then, by definition, $xy^{-1} \in G_k \setminus G_{k+1}$ and hence $y^{-1}x = (xy^{-1})^{-1} \in G_k \setminus G_{k+1}$, i.e. $\delta(y,x) = 2^{-k}$ as well. Moreover, if $\delta(x,y) = 2^{-k}$ and $\delta(y,z) = 2^{-j}$ then we have

$$xy^{-1} \in G_k \setminus G_{k+1} \quad \text{and} \quad yz^{-1} \in G_j \setminus G_{j+1}.$$

From this we get $xz^{-1} = xy^{-1}yz^{-1} \in G_k \cup G_j = G_{\min\{k,j\}}$. Thus

$$\delta(x,z) \leq 2^{-\min\{k,j\}} = \max\{2^{-k}, 2^{-j}\} = \max\{\delta(x,y), \delta(y,z)\}$$

hence the group (G,δ) is an ultrametric space.

Now, for every element $g \in G$ we define the mapping $\varphi_g : G \longrightarrow G$ with the action $\varphi_g : x \longmapsto xg$, $x \in G$. It is clear that for any subgroup $H \leq G$, φ_g maps a right coset H to the right coset Hg. Let us investigate the action induced by φ_g on the tree $T(\Sigma)$. Assume $G_i = G_{i+1} \cup G_{i+1}h_1 \cup \ldots$. be the coset partition of G_i with respect to G_{i+1}. Then the coset $G_i g_j$ is adjacent to $G_{i+1}hg_j$ and hence for an arbitrary $g \in G$ the images $\varphi_g(G_i g_j) = G_i g_j g$ and $\varphi_g(G_{i+1}hg_j) = G_{i+1}hg_j g$ are adjacent as well. Thus φ_g induces an automorphism $\overline{\varphi}_g$ of the coset tree $T(\Sigma)$. The described correspondence of elements of G and automorphisms of $T(\Sigma)$ defines an embedding:

$$\theta : \ G \hookrightarrow \operatorname{Aut} T(\Sigma), \quad \theta(g) = \overline{\varphi}_g, g \in G.$$

Theorem 4.1. *If the series Σ of subgroups $G = G_1 \supseteq G_2 \supseteq G_3 \supseteq \ldots$ is normal, then the embedding $\theta : G \hookrightarrow \operatorname{Aut} T(\Sigma)$ is continuous and we have*

$$G \hookrightarrow \mathop{\wr}_{i=1}^{\infty} G_i/G_{i+1}.$$

Proof.

(i) We show that embedding θ is a continuous map of one metric space to the other. Assume that $g_0 \in G$ and $\frac{1}{2^k} < \epsilon < \frac{1}{2^{k-1}}$. Let g be an element of G such that

$$\delta(g, g_0) \leq \varepsilon,$$

i.e. $gg_0^{-1} \in G_k \setminus G_{k+1}$. Consider the action of $\theta(g_0)$ and $\theta(g)$ on the tree $T(\Sigma)$. Since G_i is normal in G we have:

$$gg_o^{-1} \in G_i \quad \Rightarrow \quad G_i x g g_o^{-1} = G_i x \quad \Rightarrow \quad G_i x g = G_i x g_0,$$

i.e. $\theta(g)$ and $\theta(g_0)$ act in the same way on the first k levels of $T(\Sigma)$. It follows that

$$d(\theta(g_0), \theta(g)) \leq \frac{1}{2^k} < \varepsilon.$$

Thus the embedding θ is continuous.

(ii) If $g \in G$ then $\theta(g) = \overline{\varphi}_g$ is an automorphism of the tree $T(\Sigma)$. Hence we have an embedding

$$\theta : \; G \hookrightarrow \operatorname{Aut} T(\Sigma) = \wr_{i=1}^{\infty} Sym(G_i/G_{i+1}.$$

In particular, a right coset $G_{i+1}x \subset G_i$ is mapped onto the right coset $G_{i+1}xg \subset G_i g$. Thus we have

$$\theta(G) \subset \wr_{i=1}^{\infty} G_i/G_{i+1}$$

and hence there exists an embedding

$$G \hookrightarrow \wr_{i=1}^{\infty} G_i/G_{i+1}.$$

□

The above theorem is a generalization of a famous result of Kaluzhnin and Krasner [16, 17]. It yields an important universal property of the automorphism groups $\operatorname{Aut} T$ of level-homogeneous trees :

Corollary 4.2.

(i) *Every finitely generated residually finite group embeds in* $\operatorname{Aut} T$ *for some level-homogeneous tree* T*.*

(ii) *Every profinite group of countable weight embeds in* $\operatorname{Aut} T$ *for some level-homogeneous tree* T.

In fact one can observe even more.

Corollary 4.3. *Let* $\overline{\kappa} = (n_1, n_2, ...)$ *be an increasing sequence of indices. Every finitely generated residually finite group and every profinite group of countable weight embeds in* $\operatorname{Aut} T_{\overline{\kappa}}$.

Proof. Let $\overline{\kappa} = (n_1, n_2, ...)$ be the sequence as required, and let G be a finitely generated residually finite group with the descending series of normal subgroups (1). Assume $[G_i : G_{i+1}] = m_i$, $i \in \mathbb{N}$. Then it is possible to choose a subsequence $\overline{\kappa}' = (n_{i_1}, n_{i_2}, ...)$ of $\overline{\kappa}$ such that for every $j \in \mathbb{N}$ we have $n_{i_j} \geq m_j$. Then, by Theorem 4.1, group G embeds in $\operatorname{Aut} T_{\overline{\kappa}'}$, hence also in $\operatorname{Aut} T_{\overline{\kappa}}$. □

If G is a p-group then one finds a sequence Σ of the form (1) with cyclic factors of order p, i.e. $[G_i : G_{i+1}] = p$, $i \in \mathbb{N}$. Groups possessing such series of subgroups were called p_∞ – groups by L. Kaluzhnin [12, 15]. The coset tree $T(\Sigma)$ in this case is a p-adic rooted tree and we have:

Corollary 4.4. *[12] Every* p_∞*-group is embeddable in* P_∞.

From this we have in particular

Corollary 4.5.

(i) P_∞ *is universal by embedding in the class of finitely generated residually-p groups.*

(ii) P_∞ *is universal by embedding in the class of pro-p groups of countable weight.*

Remark 4.6. The group P_∞ is not the unique group with the universal property described in Corollary 4.5. In [12] the author gives as the simplest example the direct product $G = P_\infty \times C_p$ of P_∞ with a cyclic group of order p. Then G has the universal property as it contains P_∞. However, it is not isomorphic to P_∞ since G has a nontrivial center, while P_∞ is centerless.

5. Elements of P_∞ as infinite sequences of reduced polynomials

Following Kaluzhnin's ideas in [13] we introduce a practical notation for elements of the Sylow p-subgroup P_∞. Namely, every $u \in P_\infty$ may be represented as an infinite sequence (tableau) of reduced polynomials, namely:

$$u = [a_1, a_2(x_1), a_3(x_1, x_2), ...],$$

where $a_1 \in \mathbb{Z}_p$, $a_n(x_1, x_2, ..., x_{n-1}) \in \mathbb{Z}_p[x_1, x_2, ..., x_{n-1}]/\langle x_1^p - x_1, ..., x_{n-1}^p - x_{n-1}\rangle$ for $n \geq 2$. Here $\langle x_1^p - x_1, ..., x_{n-1}^p - x_{n-1}\rangle$ denotes the ideal of $\mathbb{Z}_p[x_1, x_2, ..., x_{n-1}]$ generated by $x_i^p - x_i$, $1 \leq i < n$.

For example, the identity automorphism is represented by a table with all coordinates equal to zero. If the first $s-1$ coordinates in the table u are zeros, and the s-th coordinate is nonzero, then u is said to have depth s.

For every automorphism in P_∞ defined by the portrait, one can find its corresponding table representation applying the following procedure.

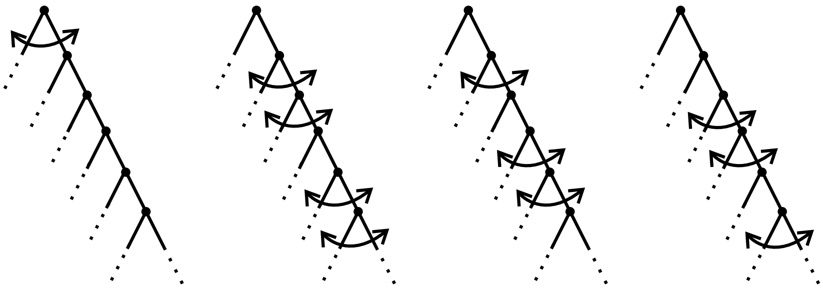
Let $f \in P_\infty$ be an automorphism of the tree $T_p(v)$, given by its portrait. First we label every vertex of the level L_k with a uniquely assigned base p number with k digits in such a way that the ancestor in level L_{k-r} of a given vertex $v \in L_k$ is assigned a base p number being the first $k-r$ digits of the label of v. At given level, f acts on descendants of a vertex by permutation. Hence the action of f on level L_{k+1} is given by a sequence of p^k permutations, all of these being powers of a fixed p-cycle $\alpha \in S_p$. The portrait on level L_{k+1} can be characterized by the sequence $(\alpha^{s_1}, \alpha^{s_2}, ..., \alpha^{s_{p^k}})$, or simply the sequence of exponents $(s_1, s_2, ..., s_{p^k})$, $0 \leq s_i < p$. Thus the action of f on the $k+1$-st level of $T_p(v)$ is determined by a function, which assigns to every vertex v_i in L_k the respective exponent s_i. In particular, we define this function as $f_k : \mathbb{Z}_{|}^{p^k} \longrightarrow Z_p$. It is known that every such function can be thought as a reduced polynomial over $\mathbb{Z}_p$. Then f is represented by the table $f = [f_1, f_2, ...]$.

For instance, we construct the tables of the generators of the (first) Grigorchuk group [5]. The group is generated by the four automorphisms of the binary tree T_2: a, b, c and d with portraits presented in Fig. 3.

The tables that correspond to automorphisms of T_2 consist of polynomials over $\mathbb{Z}_2$ reduced by the ideals $\langle x_1^2 - x_1, ..., x_n^2 - x_n\rangle$. In particular, the generators a, b, c and d can be presented as follows:

$$\begin{aligned}
a &= [1, 0, 0, 0, 0, 0, \ldots],\\
b &= [0, \alpha_2(\overline{x_1}), \alpha_3(\overline{x_2}), 0, \alpha_5(\overline{x_4}), \alpha_6(\overline{x_5}), 0, \ldots],\\
c &= [0, \alpha_2(\overline{x_1}), 0, \alpha_4(\overline{x_3}), \alpha_5(\overline{x_4}), 0, \alpha_7(\overline{x_6}), \ldots],\\
d &= [0, 0, \alpha_3(\overline{x_2}), \alpha_4(\overline{x_3}), 0, \alpha_6(\overline{x_5}), \alpha_7(\overline{x_6}), \ldots],
\end{aligned}$$

where $\alpha_{n+1}(\overline{x_n}) = \alpha_{n+1}(x_1, x_2, \ldots, x_n) = x_1 x_2 ... x_{n-1}(x_n + 1)$.



(a) Portrait of automorphism a. (b) Portrait of automorphism b. (c) Portrait of automorphism c. (d) Portrait of automorphism d.

Fig. 3: Portraits of generators of the first Grigorchuk group.

For every $n \in \mathbb{N}$ the reduced polynomial $a_n(x_1, x_2, ..., x_{n-1})$ is a sum of monomials of the type $m(x_1, x_2, ..., x_{n-1}) = a x_1^{\epsilon_1} x_2^{\epsilon_2} ... x_{n-1}^{\epsilon_{n-1}}$, where $a \in \mathbb{Z}_p^*$ and $0 \leq \epsilon_i < p$, $i = 1, 2, ..., n-1$. Given a nonzero monomial $m(x_1, x_2, ..., x_{n-1}) \in \mathbb{Z}_p[x_1, ..., x_{n-1}]/\langle x_1^p - x_1, ..., x_{n-1}^p - x_{n-1}\rangle$ we define its height $h(m)$ to be equal

$$h(m) = 1 + \epsilon_1 + \epsilon_2 \cdot p + ... + \epsilon_{n-1} \cdot p^{n-2},$$

and we set $h(0) = 0$. Then for any reduced polynomial $a(x_1, x_2, ..., x_{n-1}) \in \mathbb{Z}_p[x_1, ..., x_{n-1}]/\langle x_1^p - x_1, ..., x_{n-1}^p - x_{n-1}\rangle$ we define the height $h(a)$ to be the greatest height of all component monomials in a.

The table notation of elements of P_∞ is especially useful for computations in the group. In particular, if $u, v \in P_\infty$ are automorphisms represented by the tables

$$u = [a_1, a_2(x_1), a_3(x_1, x_2), ...], \qquad v = [b_1, b_2(x_1), b_3(x_1, x_2), ...],$$

then their product uv is given by the table $uv = [c_1, c_2(x_1), c_3(x_1, x_2), ...]$ with coordinates

$$c_s(x_1, ..., x_{s-1}) = a_s(x_1, x_2, ..., x_{s-1}) + b_s\left((x_1, x_2, ..., x_{s-1})^u\right), s \geq 1,$$

where $(x_1, x_2, ..., x_{s_1})^u = (x_1 - a_1, x_2 - a_2(x_1), ..., x_{s-1} - a_{s-1}(x_1, ..., x_{s-2}))$. In the following by u_i, $i \geq 1$, we denote the beginning of length i of the table u, i.e. $u_i = [a_1, a_2(x_1), a_3(x_1, x_2), ..., a_i(x_1, ..., x_{i-1})]$.

Below we list some results obtained by L. Kaluzhnin in [13] for the group P_n of tables with n coordinates. The same arguments suffice to show the analogous properties of infinite tables from P_∞.

Lemma 5.1.

(i) *For every reduced polynomial* $f(x_1,...,x_s)$ *and every table* $u \in P_{s+1}$ *the inequality holds:*

$$h(f - f^u) < h(f).$$

In particular, if $h(f) = k$ *there exists at least one element* $u \in P_{s+1}$ *such that the polynomial* $f(x_1,...,x_s) - f((x_1,...,x_s)^u)$ *has height equal to* $k-1$.

(ii) *For every reduced polynomial* $f(x_1,...,x_s)$ *and every table* $u \in P_{s+1}$ *of depth* r *the following inequality holds:*

$$h(f - f^u) \leq p^s - p^r.$$

In particular, for every f *there exists at least one element* $u \in P_{s+1}$ *of depth* r *such that the polynomial* $f(x_1,...,x_s) - f((x_1,...,x_s)^u)$ *has height equal to* $p^s - p^r$.

(iii) *Given an element* $u \in P_{s+1}$ *and a reduced polynomial* $u(x_1,...,x_s) \in \mathbb{Z}_p[x_1,...,x_s]/\langle x_1^p - x_1,...,x_s^p - x_s\rangle$ *the equation*

$$f(x_1,...,x_s) - f((x_1,...,x_s)^u) = g(x_1,...,x_s)$$

has a solution $f \in \mathbb{Z}_p[x_1,...,x_s]/\langle x_1^p - x_1,...,x_s^p - x_s\rangle$ *and* $u \in P_{s+1}$ *if and only if the sum of values of* $g(x_1,...,x_s)$ *is zero in every orbit of* u.

In the following $[u]_i$, $1 \leq 1 < n$, denotes the i-th coordinate of the table $u \in P_n$.

Lemma 5.2. *Let* u *and* v *be two elements of* P_{s+1} *given by*

$$\begin{aligned} u &= [a_1, a_2(x_1), ..., a_{s+1}(x_1, x_2, ...x_s)] \\ v &= [b_1, b_2(x_1), ..., b_{s+1}(x_1, x_2, ..., x_s)]. \end{aligned}$$

(i) *The conjugate* $u^v = vuv^{-1}$ *is represented by the table with coordinates given as follows:*

$$\begin{aligned} [u^v]_i &= a_i((x_1,...,x_{i-1})^{v_i} + \\ &\quad + b_i(x_1,...,x_{i-1}) - b_i((x_1,...,x_{i-1})^{u_i^v}). \end{aligned}$$

(ii) *For every table* $u \in P_{s+1}$ *of depth* r *there exists at least one table* $v \in P_{s+1}$ *such that*

$$h([vuv^{-1}]_i) \geq p^{i-1} - p^r \quad \text{for every} \quad i \geq r+1.$$

(iii) *The commutator* $[u,v]=uvu^{-1}v^{-1}$ *is represented by the table with coordinates given as follows:*

$$\begin{aligned}[uvu^{-1}v^{-1}]_i &= a_i((x_1,...,x_{i-1}) - a_i((x_1,...,x_{i-1})^{v_i^u})+ \\ &\quad +b_i((x_1,...,x_{i-1})^{u_i})+ \\ &\quad -b_i((x_1,...,x_{i-1})^{(uvu^{-1}v^{-1})_i}).\end{aligned}$$

(iv) *The height of each coordinate of the commutator* $[u,v]$ *satisfies the inequality:*

$$h([u,v]_i) < \min\{h([u]_i), h([v]_i)\}$$

or it is equal to 0. For every element $u \in P_{s+1}$ *there exists at least one element* $v \in P_{s+1}$ *such that*

$$h([u,v]_i) = h([u]_i) - 1.$$

Lemmata 5.1 and 5.2 can be easily verified by direct calculations. For detailed proofs of both lemmas we refer the reader to [13].

6. Ideal subgroups of P_∞

Using the concept of height in the set of all sequences in P_∞ we introduce a partial order $\preceq$ on P_∞ as follows. For any two elements $u=[a_1,a_2(x_1),...]$ and $v=[b_1,b_2(x_1),...]$ from P_∞ we set

$$u \preceq v \quad \Leftrightarrow \quad h(a_i) \le h(b_i) \text{ for all } i \in \mathbb{N}.$$

One verifies directly that $\preceq$ is a partial order in P_∞.

We recall that an ideal of a partially ordered set $(X,\le)$ is a nonempty subset $I \subset X$ satisfying the following two conditions:

(i) For every $x \in I$, if $y \le x$ then $y \in I$;

(ii) For every $x,y \in I$ there exists $z \in I$, such that $x \le z$ and $y \le z$.

Given an infinite sequence of nonnegative integers $\overline{h}=(h_i)_{i=1}^\infty$, $0 \le h_i < p^i$, in P_∞ we consider the ideal subset

$$P(\overline{h}) = \{u \in P_\infty \quad | \quad h(a_i) \le h_i,\ i \in \mathbb{N}\}.$$

It is easy to see that $P(\overline{h})$ is a subgroup of P_∞, which we call the ideal subgroup of P_∞. We note that ideal subgroups were originally introduced by L. Kaluzhnin in [13], who called them parallelotopic subgroups.

We present an alternative construction of the ideal subgroups. Let H be an ideal subgroup, and let u be one of the maximal tables with respect to $\preceq$ in H. By H_n we denote the subgroup of P_∞, containing all tables which have a unique nonzero coordinate in the n-th place and its height does not exceed the height of $[u]_i$:

$$H_n = \{w \in P_\infty \mid [w]_i = 0 \ \wedge \ h([w]_i) \leq h([u]_i) \text{ for } i \neq n\}.$$

Then $H = \overline{\prod_{n=1}^{\infty} H_n}$, i.e. H is the closure of the direct product of subgroup H_n, $n \in \mathbb{N}$.

Theorem 6.1.

(i) *Every ideal subgroup is closed in the profinite topology on* P_∞.

(ii) *All ideal subgroups of* P_∞ *constitute a distributive sublattice in the lattice of all subgroups of* P_∞.

Proof.

(i) Observe first that P_∞ is a topological group with the topology inherited from $\operatorname{Aut} T_p$. The basis of neighborhoods of the identity in this topology consists of subgroups $\mathcal{D}_s$, $s \in \mathbb{N}$, defined as follows:

$$\mathcal{D}_s = \{u \in P_\infty \mid u_i = 0 \text{ for } i = 1, 2, ..., s\}. \tag{2}$$

All the subgroups $\mathcal{D}_s$ and their translations $f \cdot \mathcal{D}_s$, $f \in P_\infty$ are open. Moreover, the subgroup $\mathcal{D}_s$ is the stabiliser of the s-th level of the tree T_p.

Now, if $P(\overline{h})$ is an ideal subgroup of P_∞ defined by an infinite sequence $\overline{h} = \{h_i\}_{i=1}^{\infty}$, then

$$P(\overline{h}) = P_\infty \setminus \bigcup_{s=1}^{\infty} \bigcup_{f_s^* \in P_\infty} f_s^* \cdot \mathcal{D}_s,$$

where f_s^* is a table from P_∞ such that there exist a coordinate f_i, $1 \leq i \leq s$ with $h(f_i) > h_i$. Thus $P(\overline{h})$ is closed.

(ii) Let $\overline{h} = (h_i)_{i=1}^{\infty}$ and $\overline{h'} = (h'_i)_{i=1}^{\infty}$, $0 \leq h_i, h'_i < p^i$, be two infinite sequences of nonnegative integers, and let $P(\overline{h})$ and $P(\overline{h'})$ be the respective ideal subgroups defined by these sequences. Observe that if $h \preceq h'$ then every element of $P(\overline{h})$ is contained in $P(\overline{h'})$ by the definition of the ideal. Thus $P(\overline{h}) \leq P(\overline{h'})$. Moreover, in any case we have that $h \vee h' = t$ for $t = \{t_i\}_{i=1}^{\infty}$, where $t_i = \max\{h_i, h'_i\}$, $i \in \mathbb{N}$; and $h \wedge h' = m$ for $m = \{m_i\}_{i=1}^{\infty}$, where $m_i = \min\{h_i, h'_i\}$, $i \in \mathbb{N}$. Thus the second statement of the theorem follows.

□

The normality criterion obtained in [13] for ideal subgroups of the finite groups P_m translates easily to the subgroups of P_∞:

Theorem 6.2. *[14] An ideal subgroup $P(\overline{h})$ of depth r, defined by an infinite sequence of nonnegative integers $\overline{h} = (h_i)_{i=1}^{\infty}$ is normal in P_∞ if and only if for every $i \in \mathbb{N}$ the following inequality is satisfied:*

$$h_i \geq p^{i-1} - p^r,$$

for all $i \geq r+1$.

Proof. Let $H = P(\overline{h})$ be a normal ideal subgroup of P_∞ of depth r, and let $\mathcal{D}_s$, $s \in \mathbb{N}$ be the subgroups defined in (2). Direct calculations show that for every $s \in \mathbb{N}$ the subgroup $\mathcal{D}_s$ is normal in P_∞ and hence $H \cdot \mathcal{D}_s$ is normal in P_∞ as well. Therefore $H \cdot \mathcal{D}_s/\mathcal{D}_s$ is normal in $P_\infty/\mathcal{D}_s \cong P_s$ of depth r. In [13] the author characterized all normal ideal subgroups of P_s as ideal subgroups $P(h_1, h_2, ..., h_s) \leq P_s$ with $h_i \geq p^{i-1} - p^r$, where r is the depth of the ideal subgroup. Therefore we have $H \cdot \mathcal{D}_s/\mathcal{D}_s \cong P(h_1, h_2, ..., h_s)$ for every $s \in \mathbb{N}$ and thus $H = P(\overline{h})$ with $h_i \geq p^{i-1} - p^r$, $i \in \mathbb{N}$.

Conversely, assume $H = P(\overline{h})$ to be an ideal subgroup of depth r defined by a sequence $\overline{h} = (h_i)_{i=1}^{\infty}$ satisfying $h_i \geq p^{i-1} - p^r$. Then clearly $H \cdot \mathcal{D}_s/\mathcal{D}_s \cong P(h_1, h_2, ..., h_s)$ is a normal subgroup of $P_\infty/\mathcal{D}_s$, and therefore $H \cdot \mathcal{D}_s \trianglelefteq P_\infty$ for every $s \in \mathbb{N}$. Thus the intersection

$$\bigcap_{s \in \mathbb{N}} H \cdot \mathcal{D}_s = H$$

is also normal in P_∞. □

7. Lower central series of P_∞

Knowing the form of commutators (see Lemma 5.2), it is possible to characterize all terms of the lower central series in P_∞. Namely

Theorem 7.1. *[14] The k-th term $\gamma_k(P_\infty)$ of the lower central series in P_∞ coincides with a normal ideal subgroup $P(\overline{h})$ defined by the sequence $\overline{h} = (h_i)_{i=1}^{\infty}$, where $h_i = \max\{0, p^{i-1} - k\}$.*

Proof. For $k = 1$ the description is valid. Assume then that the theorem holds for all terms of the lower central series up till the k-th term. Consider the $k+1$-st term $\gamma_{k+1}(P_\infty)$. By definition $\gamma_{k+1}(P_\infty)$ is generated by the commutators of the form $[u, v]$, where $u \in \gamma_k(P_\infty)$ and $v \in P_\infty$. Take $u = [0, 0, ..., 0, a_i(x_1, ..., x_{i-1}), 0, 0, ...]$. It follows from Lemma 5.2 and our inductive assumption that

$$h([u, v]_i) < h(u) \leq \max\{0, p^{i-1} - k\},$$

hence $h([u, v]_i) \leq \max\{0, p^{i-1} - (k+1)\}$. Moreover, there always exists an element $v \in P_\infty$ such that if only $[u, v]_i \neq 0$ then $h([u, v]_i) = p^{i-1} - (k+1)$. Therefore $\gamma_{k+1}(P_\infty)$ contains every subgroup

$$H_i = \{w \in P_\infty \mid h([w]_i) \leq p^{i-1} - (k+1) \wedge \ [w]_j = 0 \ \text{ for } j \neq i\},$$

and thus every product of elements of this type. It follows that $\gamma_{k+1}(P_\infty) \supseteq \prod_{n=1}^{\infty} H_n$ and, since $\gamma_{k+1}(P_\infty)$ is closed in P_∞, we have

$$\gamma_{k+1}(P_\infty) = \overline{\prod_{n=1}^{\infty} H_n} = P(\overline{h}),$$

where $\overline{h} = (h_i)_{i=1}^{\infty}$, where $h_i = \max\{0, p^{i-1} - k\}$. □

The above theorem is analogous to the respective characterization of the lower central series in groups P_n, $n \in \mathbb{N}$ given in [13] by L. Kaluzhnin. In particular, the derived subgroup $P'_\infty = \gamma_2(P_\infty) = [P_\infty, P_\infty]$ is the ideal subgroup $P(\overline{h})$ defined by the sequence

$$\overline{h} = (0, p-2, p^2-2, p^3-2, \ldots, p^{i-1}-2, \ldots).$$

From Theorem 7.1 we deduce a couple of observations.

Corollary 7.2. *Every factor group $\gamma_k(P_\infty)/\gamma_{k+1}(P_\infty)$ is a continual elementary abelian group.*

Proof. One verifies directly, that every element of the factor group

$$\gamma_k(P_\infty)/\gamma_{k+1}(P_\infty)$$

is an infinite sequence $(a_1, a_2, ...)$ where $a_i \in \mathbb{Z}_p$ for $i \in \mathbb{N}$. Moreover, the group operation for such sequences is simply the coordinate-wise addition. Hence $\gamma_k(P_\infty)/\gamma_{k+1}(P_\infty) \cong \mathbb{Z}_p^\infty$, as stated. □

Corollary 7.3. *P_∞ is infinitely generated as a topological group.*

Proof. Let us observe that P_∞/P'_∞ is a topological group with the quotient topology induced from P_∞. As $P_\infty/P'_\infty \cong \mathbb{Z}_p^\infty$ then P_∞/P'_∞ is not finitely generated as a topological group. The statement of the corollary follows. □

8. Isometrically characteristic subgroups of P_∞

For the discussion on characteristic subgroups, we first recall some known results on automorphism group of P_∞. The groups of automorphisms of the groups P_n and P_∞ were investigated in [3, 18, 19, 20]. In the paper [20] the author characterizes all isometric automorphisms of the group P_∞, i.e. automorphisms preserving the basic sets $\mathcal{D}_s$. In order to formulate the main result of the recalled paper we first introduce the notion of a scalar automorphism.

Let $\bar{w} = \{w_1, w_2, ...\}$ be an infinite sequence of integers, $w_i \in \mathbb{Z}_p$, $i \in \mathbb{N}$. The automorphism ω of P_∞, defined by the rule:

$$\omega([a_1, a_2(x_1), ...]) = [w_1 a_1, w_2 a_2(x_1 w_1^{-1}), w_3 a_3(x_1 w_1^{-1}, x_2 w_2^{-1}), ...]$$

for any $[a_1, a_2(x_1), ...] \in P_\infty$, is called a scalar automorphism. We denote the set of all scalar automorphisms in P_∞ by Ω_∞. If $p = 2$ then Ω_∞ is trivial. The subgroup of all inner automorphisms of P_∞ will be denoted by $\operatorname{Inn} P_\infty$.

Lemma 8.1. *[20] The subgroup* $\operatorname{Aut}_{is}(P_\infty)$ *of isometric automorphisms of* P_∞ *is decomposable into a general product of subgroups of inner and scalar automorphisms:*

$$\operatorname{Aut}_{is}(P_\infty) = \Omega_\infty \cdot \operatorname{Inn} P_\infty.$$

In this section we consider subgroups of P_∞, which are invariant to the isometric automorphisms. Subgroups of this kind were first discussed by L. Kaluzhnin in the paper [14]. We recall here the results and include a proof.

Theorem 8.2. *[14] Let* $p \neq 2$. *Then:*

(i) *Every normal ideal subgroup of* P_∞ *is invariant to all isometric automorphisms.*

(ii) *Every subgroup of* P_∞ *which is invariant to all isometric automorphisms, is a normal ideal subgroup.*

Proof.

(i) Let $H = P(\overline{h})$ to be a normal ideal subgroup of depth r of P_∞. From Theorem 6.2 it follows that H is defined by a sequence $\overline{h} = (h_i)_{i=1}^{\infty}$ such that for every $i \geq r$ we have $h_i \geq p^{i-1} - p^r$. It is enough to check the invariancy of H with respect to scalar automorphisms form Ω_∞.

Let $\omega \in \Omega_\infty$ be a scalar automorphism defined by a sequence of scalars $\bar{w} = \{w_1, w_2, ...\}$, $w_i \in \mathbb{Z}_p$, $i \in \mathbb{N}$. Then, by definition, for every table $u = [a_1, a_2(x_1), a_3(x_1, x_2), ...] \in P_\infty$ we have

$$\omega([a_1, a_2(x_1), ...]) = [w_1 a_1, w_2 a_2(x_1 w_1^{-1}), w_3 a_3(x_1 w_1^{-1}, x_2 w_2^{-1}), ...],$$

and hence $h(\omega(u)) = h(u)$. Thus $\omega(u) \in H$, as stated.

(ii) Let H be a subgroup of P_∞ which is invariant to all isometric automorphisms and let $\alpha \in \mathrm{Aut}_{is} P_\infty$. Then, as $\mathcal{D}_s$ for $s \in \mathbb{N}$ are invariant to isometric automorphisms, then $\alpha(H \cdot \mathcal{D}_s) \subset H \cdot \mathcal{D}_s$. Moreover, α induces an automorphism $\bar{\alpha}$ of $P_\infty/\mathcal{D}_s \cong P_s$ which by its isometric property, maps table

$$u = [a_1, a_2(x_1), a_3(x_1, x_2), ..., a_s(x_1, ..., x_s)] \in H \cdot \mathcal{D}_s/\mathcal{D}_s$$

to a table $\bar{\alpha}(u) \in H \cdot \mathcal{D}_s/\mathcal{D}_s$. Hence, $H \cdot \mathcal{D}_s/\mathcal{D}_s$ is isomorphic to a subgroup H_s of P_s invariant to $\bar{\alpha}$ for every $\bar{\alpha}$ induced by an isometric automorphism of P_∞. By the results of L. Kaluznin from [13] we deduce that H_s is a normal ideal subgroup of P_s. It follows that both subgroups $H \cdot \mathcal{D}_s$ and H are normal ideal subgroups of P_∞.

□

From the above theorem and Theorem 6.1 it follows directly:

Corollary 8.3. *If $p \neq 2$ then every subgroup of P_∞ which is invariant to all isometric automorphisms is closed.*

Whether there exist characteristic subgroups of P_∞ which are not closed remains an interesting question to investigate, posed by Kaluzhnin in [14].

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