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DIFFERENTIAL EQUATION OF AN LTV SYSTEM WITH FEEDBACK
RÓWNANIE RÓŻNICZKOWE UKŁADU LTV ZE SPRZĘŻENIEM ZWROTNYM

Introduction

The transformation rules of block diagrams composed of elementary SISO blocks (single input single output) of LTI class (linear time invariant) are well known in control theory. In the continuous-time domain or discrete-time domain the algebra of transforming is based on operations $(\pm, *)$ with impulse responses of elementary sections as kernels of these operations. Operations $(\pm)$ are defined as addition or subtraction functions of real variable. Operation $(*)$ is defined as convolution of real functions. In the domain of L-transforms or F-transforms algebra is based on classical definition of operations $(\pm, \cdot)$ with arguments being transfer functions or spectral functions of elementary blocks. The connections of elementary blocks, especially of the first order sections, are often used in circuit synthesis and signal processing [2].

Rules of transformation of systems composed of elementary SISO blocks of LTV class (linear time varying) are more complex [1], [3].
Generally, algebraic operations in the time domain and in the frequency domain both in the classical sense $(\pm, \cdot)$, and using convolution algebra $(\pm, *)$ are not sufficient [1], [2] to obtain the equivalent system, fig. 1.

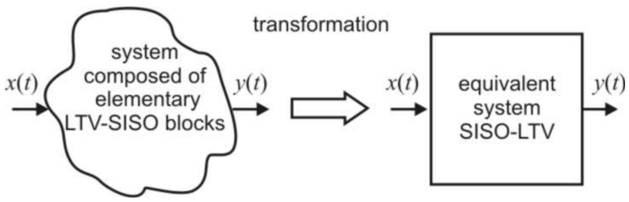

Fig. 1 Transformation of a complex LTV system into an equivalent system

This article is limited to analysis of a system with feedback composed of the first order parametric sections. The differential equation describing the LTV system with feedback has been carried out. The analysis of parallel and cascade connections of LTV sections in the time domain and in the frequency domain have been presented in works [4], [5], [6].

1. FORMALIZATION OF THE PROBLEM

The system considered in the paper consists of two first order LTV sections – the one processes signal in the feedforward direction and the second one is placed in the feedback loop. The diagram of the system has been presented in fig. 2.

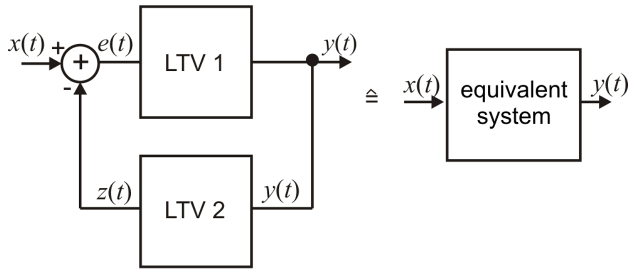

Fig. 2. Feedback system of LTV sections

Elementary LTV section has been constructed by varying the constants coefficients of the differential equation describing the stationary system. The elementary sections are described by the first order parametric differential equations (1) and (2). The variable parameters of LTV sections are referred as parametric functions $\omega_1(t)$ and $\omega_2(t)$.

The section LTV 1 is described by differential equation:

$$y'(t) + \omega_1(t)y(t) = e(t)\,. \tag{1}$$

The equation of the section LTV 2 is expressed by:

$$z'(t) + \omega_2(t)z(t) = y(t)\,. \tag{2}$$

The signal of the sum junction is specified by equation:

$$e(t) = x(t) - z(t)\,. \tag{3}$$

The solution to the differential equation (2) is determined by formula:

$$z(t) = \mathrm{e}^{-\alpha_2(t)} \int_0^t \mathrm{e}^{-\alpha_2(\tau)} y(\tau)\mathrm{d}\tau\,, \tag{4}$$

where:

$$\alpha_2(t) = \int_0^t \omega_2(t)\mathrm{d}t\,. \tag{5}$$

Substitution of the formulae (3), (4), (5) to equation (1) and subsequent differentiation results in the second order parametric differential equation:

$$y''(t) + A_1(t)y'(t) + A_0(t)y(t) = x_1(t)\,, \tag{6}$$

where the variable coefficients of the formula (6) are expressed as:

$$A_1(t) = \omega_1(t) + \omega_2(t)\,, \tag{7}$$

$$A_0(t) = \omega_1'(t) + \omega_1(t)\omega_2(t) + 1\,, \tag{8}$$

and the equivalent excitation is described by:

$$x_1(t) = x'(t) + \omega_2(t)x(t)\,. \tag{9}$$

The equation (6) describes the negative feedback system (fig. 2) composed of the first order LTV sections.

2. EXPONENTIALLY VARYING PARAMETERS IN SYSTEM WITH FEEDBACK

The considered negative feedback system of two low-pass first order LTV sections is shown in fig. 2. The exponentially variation of parametric functions has been assumed:

$$\omega_i(t) = \omega_{0i} + C_i e^{-\gamma_i t} \tag{10}$$

The parametric function of section LTV 1 is time-varying angular frequency $\omega_1(t)$, whereas the parametric function of section LTV 2 is the time-varying angular frequency $\omega_2(t)$. The waveforms of both parameters $\omega_1(t)$ and $\omega_2(t)$ are shown in fig. 3.

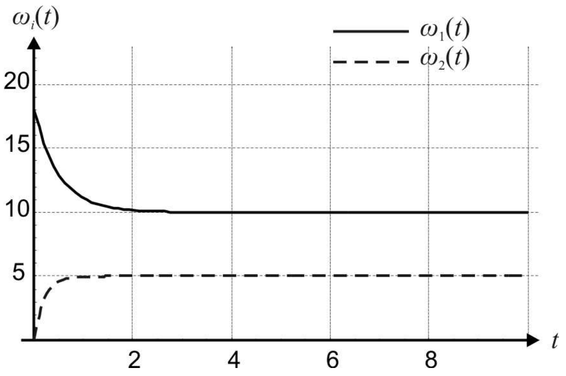

Fig. 3 Waveforms of parametric functions

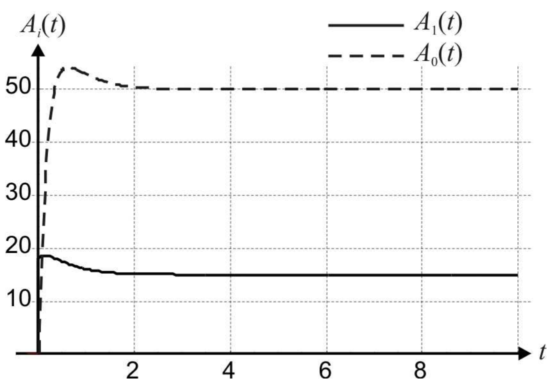

Fig. 4. Variable parameters of equivalent system

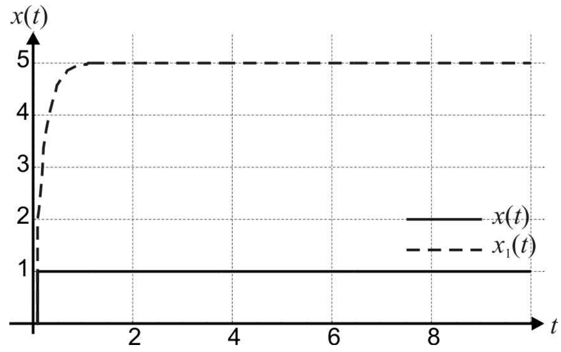

Fig. 5. Excitation $x(t)$ and equivalent excitation $x_1(t)$

The variable coefficients of differential equation of the system with feedback have been presented in fig. 4. The equivalent excitation have been shown in fig. 5.

3. CONCLUSIONS

The systems with feedback composed of parametric sections asymptotically approach the system composed of LTI sections. It is a result of assumed parametric functions. In the non-steady state of parametric function the system connections allow to form various system properties.

4. BIBLIOGRAPHY

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5. STRESZCZENIE

W artykule opisano metodę uzyskania modelu matematycznego układu LTV (linear time - varying), który składa się z dwóch połączonych sekcji LTV pierwszego rzędu z ujemnym sprzężeniem zwrotnym. Uzyskane wyniki zilustrowano przykładem.

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