# GENERALIZED FUNCTIONS AND CALCULUS OPERATORS OF *MATHEMATICA* APPLIED TO EVALUATION OF INFLUENCE LINES AND ENVELOPES OF STATICALLY INDETERMINATE BEAMS

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## Abstract

The paper presents an analytical method of finding functions of influence lines of statically indeterminate beams. There are presented solutions of a fourth order equation with a right hand side with second and third derivative of Dirac delta. There is shown that their solution are influence lines of moments and transverse forces. Moreover, thanks to Mathematica, analytical form of envelopes functions can be evaluated.

## Keywords

Influence lines, fundamental solution, generalized functions, Dirac delta, Heaviside step function, structural mechanics, differential equations, envelopes, *Mathematica*.

## 1 Introduction

Influence lines play an important role in education of structural engineers [1] and engineering practice, especially designing of bridges [4]. They are functions and called by mathematicians Green functions or fundamental solution and have important application in many engineering fields, see for example [3].

The aim of this paper is to show that thanks to *Mathematica* [7] and implemented within it generalized functions and calculus operators it is possible to propose a new analytical approach for evaluation of influence lines and envelopes of internal forces in beams, especially statically indeterminate.

The generalized function Heaviside step function [2, 6] and Dirac delta [5], implemented in Mathematica [7], are applied. Thanks to numerical experiment with equations:

$y^{(4)}(x) = \delta''(x-a)$,

$y^{(4)}(x) = \delta^{(3)}(x-a)$,

it has been found that the obtained functions looked like influence lines of simply supported beam [1] for moments and transverse forces, respectively. It is a main motivation for the presented analyses, since it can be a chance to find more general analytic solution of the problem.

First the simply supported beam has been analyzed to show that the obtained experimentally solution has a physical interpretation. Next the statically indeterminate beams has been considered. There is shown that thanks to analytical tools of Mathematica it is possible to find a close form of influence lines and envelopes of internal forces. For example influence line of bending moments in clamped-clamped beam can be evaluated with following *Mathematica* function:

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$$\mathbf{Assuming}[-1<\alpha<1\&\&-1<\xi<1,\mathbf{FullSimplify}[\mathbf{DSolve}[\{y^{(4)}[\xi]=$$
$$=\frac{1}{2}\mathbf{DiracDelta}''[\xi-\alpha]l,y[-1]==0,y[1]==0,y'[-1]=$$
$$=0,y'[1]==0\},y[\xi],\xi]]]$$
$$\left\{\left\{y[\xi]\to\frac{1}{8}l\left(-\left(1+\alpha(-2+\xi)\right)(1+\xi)^2+4(-\alpha+\xi)\mathrm{HeavisideTheta}[-\alpha+\xi]\right)\right\}\right\}$$

Graphical capabilities of Mathematica makes it possible to illustrate the problem. For example:

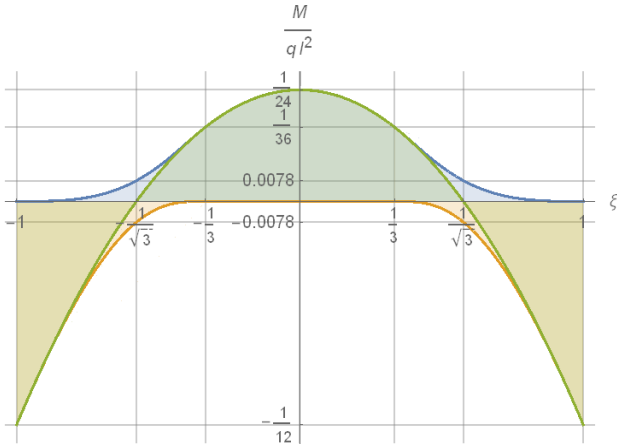


Fig. 1: $M_g(\xi)$, $M_{p+}(\xi)$, $M_{p-}(\xi)$ functions for clamped-clamped beam

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