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ANALITYCAL METHOD OF DETERMINING THE FREEZING FRONT LOCATION

Summary. In this paper an analytical method of solving the selected class of problems which can be reduced to the one-phase solidification problem of a plate with the unknown a priori, varying in time boundary of the region in which the solution is sought.

ANALITYCZNA METODA WYZNACZANIA POŁOŻENIA FRONTU KRZEPNIĘCIA

Streszczenie. W artykule przedstawiono analityczną metodę rozwiązywania wybranej klasy problemów, które można sprowadzić do jednofazowego zagadnienia krzepnięcia płyty z nieznaną a priori zmienną w czasie granicą obszaru, w którym poszukiwane jest rozwiązanie.

1. Introduction

Mathematical modeling of thermal processes combined with the reversible phase transitions of type: solid phase - liquid phase leads to formulation of the parabolic boundary problems with the moving boundary. Solving of such defined problem requires, most often, to use sophisticated numerical techniques and far advanced

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mathematical tools. Excellent illustration of the complexity of considered problems, as well as of the variety of approaches used for finding their solutions, is given in the papers [1, 3–5].

In the current paper, we present the, especially attractive from the engineer point of view, analytical method of solving the selected class of problems which can be reduced to the one-phase solidification problem of a plate with the unknown a priori, varying in time boundary of the region in which the solution is sought. Proposed method is based on the known formalism of initial expansion of the sought function, describing the temperature field, into the power series. Coefficients of this series will be determined with the aid of boundary conditions.

2. Formulation of the mathematical model

Each correctly defined mathematical model, related to the real problems, requires, at its formulation level, to determine unambiguously the physical determinants of the process concerned by the given model. Taking it into account we begin with the following initial assumptions:

— temperature of the entire system at the initial moment is equal to the phase transition temperature T^* which gives the possibility to assume the heat conduction to be a dominant mechanism of heat transfer,

— phase transition happens in the strictly determined, constant temperature T^* which is typical for the materials of the ideal dielectric properties,

— material parameters, it means: ρ – mass density, λ – thermal conductivity and c – specific heat and, from this, the thermal diffusivity coefficient a as well, are independent on temperature and equal in both phases,

— process of the heat transfer is one-dimensional and symmetric.

Taken assumptions eliminate from the consideration the liquid phase and enable to focus on the solid phase. Mathematical model of the problem, formulated in this way (one-phase Stefan problem), is defined by the following system of equations

— the heat conduction equation describing the field of temperature T, varying in time and space, in the formed solid phase

$$\frac{\partial T(x,t)}{\partial t} = a\frac{\partial^2 T(x,t)}{\partial x^2}, \quad x \in (\varphi(t), \bar{x}),\ t \in (0, t^*\rangle, \tag{1}$$

where x denotes the spatial variable, t – time variable, $\bar{x}$ – half of thickness of the plate, t^* – duration of the process and $\varphi(t)$ is a function describing the freezing front location (1) varying in time, it means

$$\varphi(t) = \bar{x} - \xi(t), \tag{2}$$

where $\xi(t)$ denotes a function describing the thickness of solidified layer, varying in time, and

$$\xi(0) = 0; \tag{3}$$

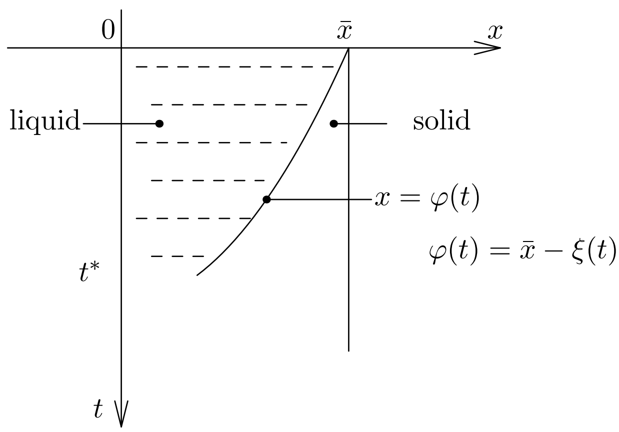

Fig. 1. Graphic illustration of the modeled process
Rys. 1. Ilustracja graficzna modelowanego procesu

— energy balance conditions in the freezing front

$$T(\varphi(t), t) = T^*, \quad t \in (0, t^*\rangle, \tag{4}$$

$$-\lambda \left.\frac{\partial T(x,t)}{\partial x}\right|_{x=\varphi(t)} = \rho\kappa\frac{d\varphi(t)}{dt}, \quad t \in (0, t^*\rangle, \tag{5}$$

where κ denotes the latent heat;

— one of the following boundary conditions on the heat transfer surface: boundary condition of the first kind

$$T(\bar{x},t) = \psi(t), \quad t \in (0, t^*\rangle, \tag{6}$$

or boundary condition of the second kind

$$-\lambda \left. \frac{\partial T(x,t)}{\partial x} \right|_{x=\bar{x}} = q(t), \quad t \in (0, t^*\rangle, \tag{7}$$

or boundary condition of the third kind

$$-\lambda \left. \frac{\partial T(x,t)}{\partial x} \right|_{x=\bar{x}} = \alpha(t)(T(\bar{x},t) - T^{\infty}), \quad t \in (0, t^*\rangle. \tag{8}$$

In the last equations, functions $\psi(t)$, $q(t)$ and $\alpha(t)$ describe, respectively, temperature of the plate surface, heat flux and the heat transfer coefficient, all varying in time ($\psi(t) \leqslant T^*$, $q(t) \geqslant 0$, $\alpha(t) \geqslant 0$), whereas T^{∞} denotes the ambient temperature.

3. Method of solution

As we have previously mentioned, method of solving the formulated above problem is based, in the first step, on the proper presentation of the function, representing the expected solution, in the form of power series. In the considered case, the series is of the following form

$$T(x,t) = \sum_{i=0}^{\infty} A_i(t) \frac{(x - \bar{x} + \xi(t))^i}{i!}, \tag{9}$$

where $A_i(t)$ denote the unknown, dependent on time, functional coefficients. We determine those coefficients by using equation (1) and conditions (4) and (5).

Relation (9) implies that

$$\frac{\partial T(x,t)}{\partial x} = \sum_{i=0}^{\infty} A_{i+1}(t) \frac{(x - \bar{x} + \xi(t))^i}{i!}, \tag{10}$$

$$\frac{\partial^2 T(x,t)}{\partial x^2} = \sum_{i=0}^{\infty} A_{i+2}(t) \frac{(x - \bar{x} + \xi(t))^i}{i!} \tag{11}$$

and

$$\frac{\partial T(x,t)}{\partial t} = \sum_{i=0}^{\infty} \left[A_i'(t)\frac{(x-\bar{x}+\xi(t))^i}{i!} + \xi'(t)A_{i+1}(t)\frac{(x-\bar{x}+\xi(t))^i}{i!} \right], \tag{12}$$

where $A_i'(t)$ and $\xi'(t)$ mean the derivative of functions $A_i(t)$ and $\xi(t)$, respectively, with respect to t.

By substituting the received formulas into equation (1) we obtain

$$\begin{gathered}\sum_{i=0}^{\infty} \left[A_i'(t)\frac{(x-\bar{x}+\xi(t))^i}{i!} + \xi'(t)A_{i+1}(t)\frac{(x-\bar{x}+\xi(t))^i}{i!} \right] = \\ = a\sum_{i=0}^{\infty} A_{i+2}(t)\frac{(x-\bar{x}+\xi(t))^i}{i!}.\end{gathered} \tag{13}$$

By comparing the terms situated by the expressions $\frac{(x-\bar{x}+\xi(t))^i}{i!}$, $i = 0, 1, 2, \ldots$, on the both sides of equation (13), we get

$$A_i'(t) + \xi'(t)A_{i+1}(t) = aA_{i+2}(t) \quad i = 0, 1, 2, \ldots. \tag{14}$$

From the conditions (4), (5) and (2) it results that

$$A_0(t) = T^*, \tag{15}$$

$$A_1(t) = -\frac{\rho\kappa}{\lambda}\xi'(t). \tag{16}$$

Since we have already the coefficients $A_0(t)$ and $A_1(t)$, we can determine the other coefficients $A_i(t)$, $i = 2, 3, \ldots$, because, by using formula (14), we have

$$A_{i+2}(t) = \frac{1}{a}(A_i'(t) + \xi'(t)A_{i+1}(t)), \quad i = 0, 1, 2, \ldots, \tag{17}$$

which implies that

$$A_2(t) = -\frac{\rho\kappa}{a\lambda}(\xi'(t))^2, \tag{18}$$

$$A_3(t) = -\frac{\rho\kappa}{a\lambda}\xi''(t) - \frac{\rho\kappa}{a^2\lambda}(\xi'(t))^3, \tag{19}$$

and so on.

Excluding the coefficient $A_0(t)$, all the other coefficients $A_i(t)$, $i = 1, 2, 3, \ldots$, will depend on the still unknown function $\xi(t)$, its powers and derivatives. One can try to determine analytically this function by using one of the conditions (6)–(8). In particular, for condition (6) we receive

$$\sum_{i=0}^{\infty} A_i(t)\frac{\xi^i(t)}{i!} = \psi(t). \tag{20}$$

However, equation (20) is as much complicated that determination of the function $\xi(t)$ with the aid of this equation is impossible. Anyway, it is possible to calculate the approximate solution by taking only two or three first components of the series situated on the left side of equation (20).

By reducing the series to only two first components we get the equality

$$A_0(t) + A_1(t)\xi(t) = \psi(t). \tag{21}$$

Hence, after substituting for $A_0(t)$ and $A_1(t)$ the values determined by relations (15) and (16), we receive the differential equation

$$T^* - \frac{\rho\kappa}{\lambda}\xi'(t)\xi(t) = \psi(t). \tag{22}$$

Assumptions accepted at the beginning of our considerations imply that the function $\xi(t)$ must take only the nonnegative values and satisfy equation (3). This information is sufficient to solve the equation (22) and to determine unambiguously the function $\xi(t)$. Thus we get

$$\xi(t) = \sqrt{\frac{2\lambda}{\rho\kappa}\int_0^t (T^* - \psi(\tau))d\tau}. \tag{23}$$

Similarly, by taking only three first components in the series situated on the left side of equation (20), we receive the relation

$$A_0(t) + A_1(t)\xi(t) + A_2(t)\frac{\xi^2(t)}{2} = \psi(t), \tag{24}$$

and, subsequently, the differential equation

$$T^* - \frac{\rho\kappa}{\lambda}\xi'(t)\xi(t) - \frac{\rho\kappa}{a\lambda}\frac{(\xi'(t)\xi(t))^2}{2} = \psi(t). \tag{25}$$

By solving differential equation (25) with condition (3) we finally obtain

$$\xi(t) = \sqrt{2a\int_0^t \left(\sqrt{1 + \frac{2c}{\kappa}(T^* - \psi(\tau))} - 1\right)d\tau}. \tag{26}$$

Although the formulas (23) and (26) determine only the approximate thickness of the solidified layer, they can be successfully used in practise. In particular case, in which $\psi(t) = T^0 = constans$, $t \in (0, t^*)$, the considered formulas take the following, very simple form

$$\xi(t) = \sqrt{\frac{2\lambda}{\rho\kappa}(T^* - T^0)t} \tag{27}$$

and

$$\xi(t) = \sqrt{2a\left(\sqrt{1+\frac{2c}{\kappa}(T^{*}-T^{0})}-1\right)t}. \tag{28}$$

Proposed method of determining the function describing the thickness of solidified layer indicates that the formulas (26) and (28) are more precise than the formulas (23) and (27). This conclusion is illustrated by the example below.

Example 1

Let us assume that the material of solidified plate of the thickness $2\bar{x} = 0.2$ [m] is specified by the following parameters: mass density $\rho = 7000$ [kg/m^3], thermal conductivity $\lambda = 100$ [W/m K], specific heat $c = 600$ [J/kg K], latent heat $\kappa = 60$ [kJ/kg], temperature of solidification $T^{*} = 800\,^{\circ}$C and that the transfer of heat with the environment is defined by the condition of the first kind (6), with the function ψ of the following form

$$\psi(t) = 900 - 100e^{at}. \tag{29}$$

Additionally, let us assume that we consider the solidification process until the moment of time $t^{*} = 4200$ s.

For the above defined data the considered problem posses the exact solution. In particular, function $T(x,t)$ describing the temperature field in the solidified part of the plate is the following

$$T(x,t) = 900 - 100e^{a^{*}t+x-\bar{x}}, \tag{30}$$

whereas the thickness of solidified layer $\xi(t)$ is defined by the relation

$$\xi(t) = a^{*}t, \tag{31}$$

where $a^{*} = a/m$ is constant of the numerical value a.

By applying the approach proposed in this paper we receive the approximate functions $T_1(x,t)$ and $T_2(x,t)$ describing the temperature field in the solidified part of the plate, as well as the approximate functions $\xi_1(t)$ and $\xi_2(t)$ defining the thickness of solidified layer, in dependence on the number of components of the series (9) being the basis of the solution.

If we reduce the series to only two first components, then we obtain

$$T_1(x,t) = T^{*} - \frac{\rho\kappa}{\lambda}\xi_1'(t)(x-\bar{x}+\xi_1(t)), \quad x \in (\bar{x}-\xi_1(t),\bar{x}), \quad t \in (0,t^{*}\rangle, \tag{32}$$

where, according to the formula (23), we have

$$\xi_1(t) = \sqrt{\frac{2\lambda}{\rho\kappa}\int_0^t (T^* - \psi(\tau))d\tau}, \quad t \in (0, t^*\rangle. \tag{33}$$

Whereas, by taking three components of the series we receive

$$\begin{gathered} T_2(x,t) = T^* - \frac{\rho\kappa}{\lambda}\xi_2'(t)(x - \bar{x} + \xi_2(t)) - \frac{\rho\kappa}{\lambda a}(\xi_2'(t))^2 \frac{(x - \bar{x} + \xi_2(t))^2}{2}, \\ x \in (\bar{x} - \xi_1(t), \bar{x}), \quad t \in (0, t^*\rangle, \end{gathered} \tag{34}$$

where, in accordance with the formula (26), we have

$$\xi_2(t) = \sqrt{2a\int_0^t \left(\sqrt{1 + \frac{2c}{\kappa}(T^* - \psi(\tau))} - 1\right) d\tau}, \quad t \in (0, t^*\rangle. \tag{35}$$

In Figure 2 graphs of the exact function $\xi(t)$ together with the approximate functions $\xi_1(t)$ and $\xi_2(t)$, describing the thickness of solidified layer, are displayed. Whereas, Figure 3 presents distributions of absolute errors received for both of the approximate solutions.

Analysis of the received results implies that formulas based on the greater number of components of the proper series give, obviously, more precise results. However, results obtained with the aid of formulas in which only two components of the series were taken are sufficiently precise as well.

Moreover, by using the formulas

$$\delta_i^{\max} = \max_{(x,t)\in\Delta_i} |T(x,t) - T_i(x,t)|, \quad i = 1, 2, \tag{36}$$

where $\Delta_i = (\bar{x} - \xi_i(t), \bar{x}) \times (0, t^*\rangle$, one can calculate the maximal absolute errors $\delta_i^{\max}$ made by applying the formulas (32) and (34) for determining the approximate temperature field. In the considered case the errors are equal to $\delta_1^{\max} = 0.233$ and $\delta_2^{\max} = 0.501$, respectively, which is 0.03% and 0.06% of the temperature field value in points of these maximums, respectively.

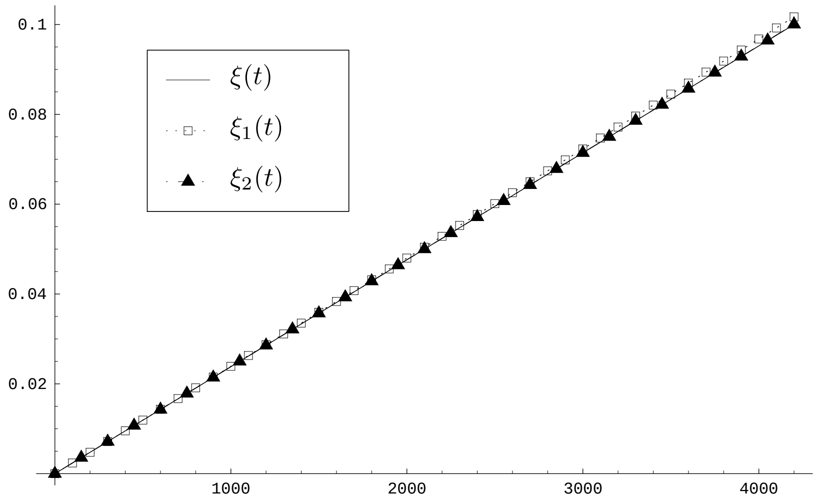

Fig. 2. Graphs of the functions describing the thickness of solidified layer
Rys. 2. Wykresy funkcji określających grubość warstwy zakrzepłej

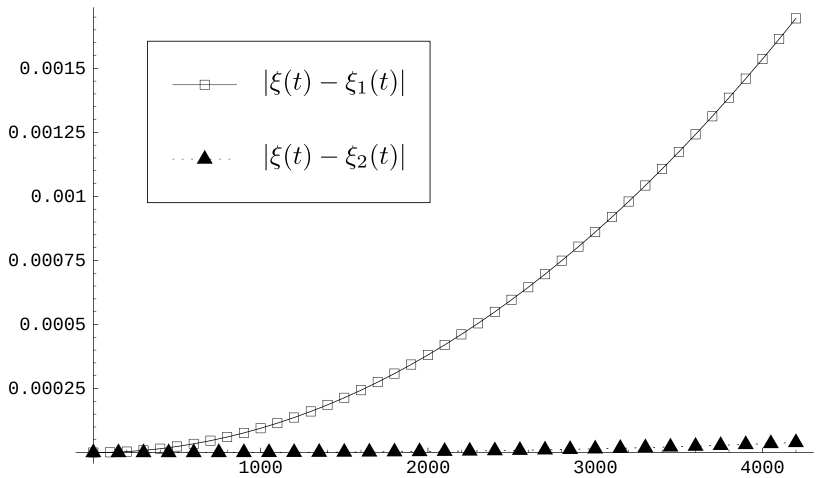

Fig. 3. Distributions of absolute errors of the approximate functions describing the thickness of solidified layer
Rys. 3. Rozkłady błędów bezwzględnych przybliżeń funkcji określających grubość warstwy zakrzepłej

4. Solutions of boundary conditions of the second and third kinds

In the previous section we were focused on the boundary condition of the first kind (6) and we developed, for this condition, the formulas enabling to determine the approximate field of temperature in the solidified part of the plate, as well as the formulas making possible to calculate the thickness of solidified layer. In this section we will repeat the procedure, but for the case of boundary conditions of the second (7) and third kinds (8).

First, let us concentrate on the boundary condition of the second kind (7). Taking into account the relations (9) and (10), the considered boundary condition implies the following equation

$$-\lambda \sum_{i=0}^{\infty} \frac{A_{i+1}(t)}{i!} (\xi(t))^i = q(t), \tag{37}$$

where the functional coefficients $A_i(t)$, =1,2,3,..., are defined by the relations (15)–(17) and they are dependent on the unknown function $\xi(t)$, its powers and derivatives. Practical determination of the function $\xi(t)$ from the equation (37) is possible only if we take just the finite number of components in the series situated on the left side of equation (37).

By reducing the series to only two first components we receive the equation

$$-\lambda \left(-\frac{\rho\kappa}{\lambda} \xi'(t) - \frac{\rho\kappa}{\lambda a} (\xi'(t))^2 \xi(t) \right) = q(t), \tag{38}$$

which, after few simple transformations, can be written in the form

$$\frac{\rho\kappa}{a} \xi(t)(\xi'(t))^2 + \rho\kappa\xi'(t) - q(t) = 0. \tag{39}$$

Assumptions accepted in Section 2 imply that function $\xi'(t)$ takes only the non-negative values. Thus, by solving equation (39) with respect to the unknown $\xi'(t)$ we obtain the differential equation

$$\xi'(t) = \frac{a}{2\xi(t)} \left(\sqrt{1 + \frac{4q(t)\xi(t)}{\rho\kappa a}} - 1 \right), \tag{40}$$

which, in case of $q(t)$ being the constant function ($q(t) = constans$), is the separable equation possessing, after using the condition (3), analytical solution in form of the implicit function

$$\left(1 + \frac{4q\xi(t)}{\rho\kappa a}\right) \left(3 + 2\sqrt{1 + \frac{4q\xi(t)}{\rho\kappa a}}\right) = \frac{24q^2}{a\rho^2\kappa^2} t + 5. \tag{41}$$

Now, let us verify the quality of reconstructed thickness of the solidified layer $\xi(t)$ for the discussed problem with boundary condition of the second kind (7). The following example will illustrate our considerations.

Example 2

We assume that the material of solidified plate is characterized by the same parameters as in Example 1, but the heat transfer with environment is defined by the boundary condition of the second kind (7), where function $q(t)$ is of the form

$$q(t) = 100\lambda e^{at}. \tag{42}$$

Function $T(x,t)$, describing the temperature field in the solidified part of the plate, as well as the function $\xi(t)$, describing the thickness of solidified layer, are obviously defined, as in Example 1, by the formulas (30) and (31), respectively.

By reducing the series, situated on the left side of equality (37), to only two components we receive, according to formula (40), the proper relation which allows to calculate numerically the sought function $\xi_1(t)$:

$$\xi_1'(t) = \frac{a}{2\xi_1(t)}\left(\sqrt{1+\frac{400\lambda e^{at}\xi_1(t)}{\rho\kappa a}}-1\right). \tag{43}$$

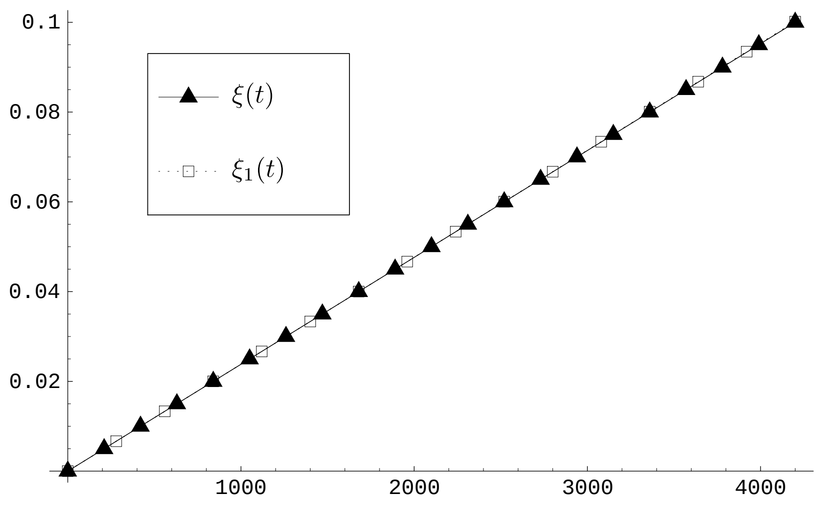

Fig. 4. Graphs of the functions describing the thickness of solidified layer
Rys. 4. Wykresy funkcji określających grubość warstwy zakrzepłej

In Figure 4 the graph of the exact function $\xi(t)$ describing the thickness of solidified layer together with its approximation $\xi_1(t)$ are displayed. Whereas, Figure 5 presents the distribution of absolute error of the approximate solution. Obtained results show that the formula based on the only two components of the series is sufficiently precise also for the problem with boundary condition of the second kind and the maximal absolute error of approximate solution is equal to just 0.00014.

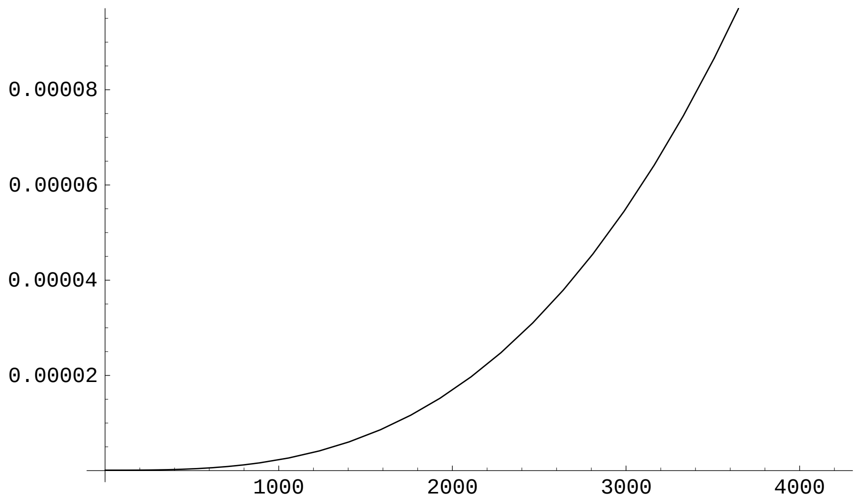

Fig. 5. Distribution of errors $|\xi(t) - \xi_1(t)|$ of approximate function describing the thickness of solidified layer

Rys. 5. Rozkład błędu $|\xi(t) - \xi_1(t)|$ przybliżenia funkcji określającej grubość warstwy zakrzepłej

Let us now proceed to boundary condition of the third kind (8) from which, by taking into account the relations (9) and (10), the following equation results

$$-\lambda \sum_{i=0}^{\infty} \frac{A_{i+1}(t)}{i!}(\xi(t))^i = \alpha(t) \left(\sum_{i=0}^{\infty} \frac{A_i(t)}{i!}(\xi(t))^i - T^{\infty} \right), \tag{44}$$

where the functional coefficients $A_i(t)$, =1,2,3,..., are defined by the relations (15)–(17) and they are dependent on the unknown function $\xi(t)$, its powers and derivatives. Practical determination of the function $\xi(t)$ from the equation (44) is possible only by taking just the finite number of components in the series situated on the left side of equation (44).

By reducing the series to only two first components we receive, from the relation (44) after transformations, the following equation

$$\frac{\rho\kappa}{a}(\xi'(t))^2\xi(t) + \rho\kappa\xi'(t) = \alpha(t)\left(T^* - T^\infty - \frac{\rho\kappa}{\lambda}\xi'(t)\xi(t)\right). \tag{45}$$

Assumptions accepted in Section 2 imply that function $\xi'(t)$ takes only the non-negative values. Thus, by solving equation (45) with respect to the unknown $\xi'(t)$ we get

$$\xi'(t) = \frac{a}{2\xi(t)}\left(\sqrt{\left(\frac{\lambda + \alpha(t)\xi(t)}{\lambda}\right)^2 + \frac{4\xi(t)\alpha(t)(T^* - T^\infty)}{a\rho\kappa}} - \frac{\lambda + \alpha(t)\xi(t)}{\lambda}\right). \tag{46}$$

By solving the above equation one can calculate $\xi_1(t)$ with the aid of condition (3). Unfortunately, the solution is not expressed by means of the elementary function, however equation (46) can be solved numerically by applying, for instance, the computational software Mathematica [2].

We will consider now the problem of reconstructing the thickness of solidified layer $\xi(t)$ in case of boundary condition of the third kind (8), illustrated by the following example.

Example 3

Analogically, we assume that the material of solidified plate is specified by the same parameters as in the previous examples and that the ambient temperature $T^\infty = 30\,^\circ\mathrm{C}$, but the heat transfer with environment is defined by the boundary condition of the third kind (8), where function $\alpha(t)$ is of the following form

$$\alpha(t) = \frac{\lambda^* e^{at}}{8.7 - e^{at}}, \tag{47}$$

where $\lambda^* = \lambda/m$ is constant of the numerical value λ.

Certainly, the functions $T(x,t)$, describing the temperature field in the solidified part of the plate, and $\xi(t)$, describing the thickness of solidified layer, are defined, as in the previous examples, by the formulas (30) and (31), respectively.

By reducing the series situated on the left side of equality (44) to only two first components we receive, according to formula (46), the relation thanks to which we can numerically determine the sought function $\xi_1(t)$.

Figure 6 presents the graph of the exact function $\xi(t)$, describing the thickness of solidified layer, together with its approximation $\xi_1(t)$. While, in Figure 7 the distribution of absolute error of this approximate solution is displayed. So, we can see that even for the boundary condition of the third kind the formula based on the only two components of the series in the relation (44) is good enough to receive sufficiently precise approximate results, maximal absolute error of which is equal to just 0.00016.

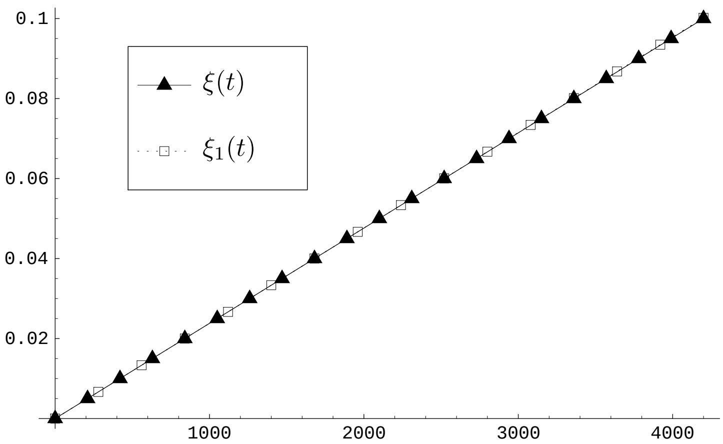

Fig. 6. Graphs of the functions describing the thickness of solidified layer
Rys. 6. Wykresy funkcji określających grubość warstwy zakrzepłej

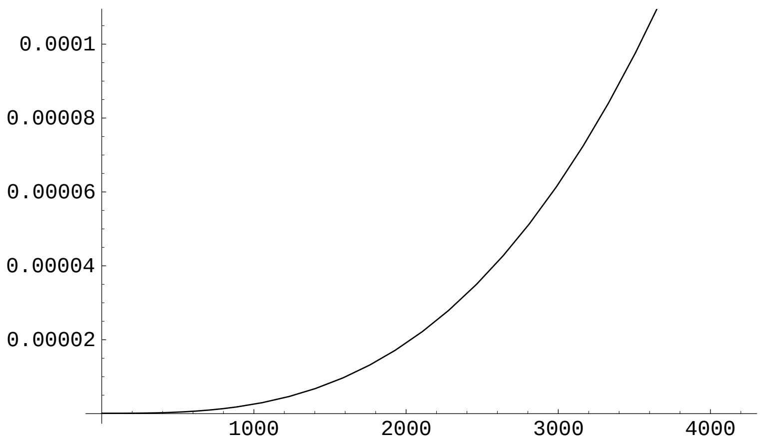

Fig. 7. Distribution of errors $|\xi(t) - \xi_1(t)|$ of approximate function describing the thickness of solidified layer
Rys. 7. Rozkład błędu $|\xi(t) - \xi_1(t)|$ przybliżenia funkcji określającej grubość warstwy zakrzepłej

5. Conclusions

The paper presents the analytical method of approximate solving the selected kind of problems which can be reduced to the one-phase solidification problem of a plate with the unknown a priori, varying in time boundary of the region in which the solution is sought. Proposed method is based on the expansion of the sought function, describing the temperature field, into the power series, some coefficients of which are determined by using the boundary conditions. Proposed approach is illustrated by the examples for the each kind of boundary conditions.

By applying the presented method we receive the appropriate function series (in the form depending on the considered boundary condition) which, after reducing to the finite number of components, give the differential equations. Solutions of those equations represent the approximate solutions of the discussed problem. With regard to the rapid convergence of the obtained series, few initial components taken into consideration may assure a satisfying reconstruction of the exact solution.

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Omówienie

W artykule przedstawiono, szczególnie atrakcyjną z inżynierskiego punktu widzenia, metodę analityczną rozwiązania wybranej klasy problemów, które można sprowadzić do jednofazowego zagadnienia krzepnięcia płyty z nieznaną a priori zmienną w czasie granicą obszaru, w którym poszukiwane jest rozwiązanie. Metoda ta bazuje na znanym formalizmie wstępnego rozwinięcia poszukiwanej funkcji, opisującej pole temperatury, w szereg potęgowy, którego pewne współczynniki wyznaczane są z warunków brzegowych. Zaprezentowane zastosowania zilustrowane zostały przykładami.

Stosując omawianą metodę otrzymujemy odpowiedni szereg lub szeregi funkcyjne (w zależności od rodzaju warunku brzegowego), które po obcięciu na pewnym skończonym miejscu wyznaczają równania różniczkowe. Rozwiązanie uzyskanych równań jest przybliżonym rozwiązaniem zagadnienia. Ze względu na szybką zbieżność otrzymanych szeregów, uwzględnienie kilku początkowych wyrazów zapewnia satysfakcjonujące przybliżenie rozwiązania dokładnego.