Application of the deterministic chaos in AC electric arc furnace modeling

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***Abstract*—Electric arc furnaces (EAFs) are widely used in the steel production and recycling process. However, their application is associated with various power quality problems whose mitigation requires a reliable model of the EAF. In this paper, we have proposed a model based on a power balance equation, where the coefficients are assumed to be responsible for the nonlinear and stochastic behavior of the EAF. These coefficients have been modeled with some variables derived from a chaotic system, while previously developed models mostly applied chaotic systems to modulate the arc length or the voltage signal itself. The paper presents the application and evaluation of four different chaotic systems: Chua, Lorenz, Rössler, and a four-wing attractor system. The parameters of each system have been fitted to the measurement data concerning the minimal discrepancy between the distribution and autocorrelation of the output data of the model and the measurements.**

***Index Terms*—chaos theory, Chua circuit, dynamical system, electric arc furnace, Lorenz system, Rössler system**

I. Introduction

Environmental protection and sustainability issues are closely related to increased energy consumption and the depletion of conventional sources such as natural gas or coal. The development of technologies related to renewable energy sources is one of the ways to build energy security. However, other equally important actions that have been undertaken for years now are connected with the minimization of energy losses, where possible. For this reason, it is important to find the causes of such losses. One of them is related to the specific needs of various industrial facilities, which often require the use of devices that lead to power quality problems.

The electric arc furnace (EAF) is an example of a load that, due to its nonlinear and randomly changing characteristics, can cause serious power quality problems such as harmonics or voltage fluctuations [1]. These issues consequently lead to increased energy losses, for example, in transmission lines. Mitigation of such problems can be achieved through various means such as unified power quality conditioners (UPQCs) [2] or active power filters (APFs) [3], for example. However, the correct design and installation of these power quality improvement systems require a reliable EAF model [4]. This model can also improve the operation of the EAF itself, leading to a reduction in heating time or equipment wear.

Most of the EAF models developed so far are based on a deterministic foundation in the form of differential equations [5], [6] or various approximations of the measured characteristics [7], [8], [9]. In order for the model to reflect EAF's dynamic nature more properly, many researchers expand their models by the addition of another time-varying component. For simplicity, those time variations are mostly considered random and, therefore, are modeled with stochastic processes, for example, white noise or more complex processes [10]. However, the EAF is, by nature, an object where the dynamics of molten steel and plasma affect the voltage and current waveforms recorded from the electrodes' point of view. The unpredictability visible in the EAF measurement data may be related to the chaotic behavior that is distinctive for some dynamical systems. In [11] it has been proven that EAF indeed exhibits chaotic behavior in voltage fluctuations. Similar conclusions were drawn in [12], [13], and [14]. This fact justifies the application of chaos theory in EAF modeling, as a deterministic and chaotic system can seem to be stochastic if it is complex enough. This approach has been applied in many models, dealing with DC or AC EAFs. In the case of the DC EAF, the models developed so far use chaotic responses as an additive signal modulating the DC component obtained from a nonchaotic deterministic submodel. For this purpose, the Lorenz [15], Chua [16], or Rössler [17], [18] systems have been applied. AC furnace models, similarly, include an approach in which the voltage signal is modulated with the chaotic signal through addition [19], [20], [21] or other ideas in which chaos is used to modulate the parameters of the arc resistance and inductance [22]. In the case of those papers, the EAF models applied Chao chaotic circuit, logistic map, or again Lorenz and Chua systems.

The aim of this paper is to introduce a novel approach to EAF modeling based on chaos theory, where changes in the shape of the EAF characteristic v-i result from variations in the values of the coefficients of the power balance equation. Those variations are reflected by signals obtained from a chaotic system. An important advantage of the proposed approach is that the observable chaotic behavior is attributed to the existing coefficients of the electric arc model, unlike in many other cases presented in the literature. As a result, there is no need for the introduction of any new auxiliary variables lacking physical interpretation that would be used for signal modulation. The paper describes the process of application and comparison of four different chaotic systems: Chua, Lorenz, Rössler, and a four-wing attractor system introduced in [23]. Their parameters, maintaining chaotic behavior, have

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been fitted with a genetic algorithm to the measurement data. Another strength of the proposed approach is related to the fact that the system parameters are fitted to the measurement data. Most existing chaotic models omit this stage assuming one set of parameters, while our models optimize the parameters according to goal functions, allowing the best fitting not only in terms of the values distribution but also their autocorrelation. In this way, four chaotic models of the AC EAF have been developed and evaluated.

II. System description

The electrical circuit of a three-phase EAF consists of a step-down transformer and flexible cables that supply power to the graphite electrodes. The electrodes are positioned by a control system in the furnace chamber. When the height is close enough to the furnace charge, an electric arc ignites between the scrap steel and the electrode. A simplified diagram of a typical EAF supply system is presented in Fig. 1. It includes a supplying HV/MV transformer (T_1) and an EAF MV/LV transformer (T_2). Short circuit reactance is represented with X_{LSC} in bus 1. Moreover, the feeder inductance and resistance is represented with L_f and R_f, while L_c and R_c are the equivalent impedance parameters of the EAF flexible cables and graphite electrodes.

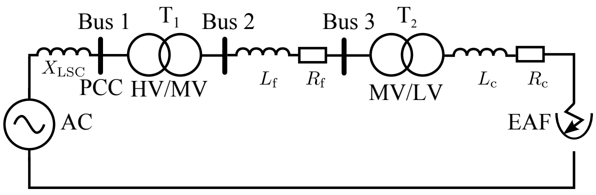

Fig. 1. An electrical circuit of the EAF.

The EAF work cycle contains stages in which the charge state varies. The stages are as follows: charging phase, melting phase, refining phase, deslagging phase, steel tapping, and furnace turnaround. Each stage has its importance in the production process, but the melting stage specifically has the greatest influence on the power system as a result of the strong distortions occurring in the electric arc phenomenon. Therefore, it is important to consider the EAF characteristics derived from this particular stage. As a basis for the EAF modeling, we used the phase voltage and current measured in an industrial EAF during the melting stage. Data preprocessing has taken into account the influence of resistance R_c and inductance L_c. Due to this fact, the data represent only the electric arc and do not include the influence of other circuit elements, e.g., flexible cables. Our measurement data consist of 10-second long EAF current and voltage recordings. As shown in Fig. 2, the waveforms are distorted in a different manner, and two different stochastic characteristics can be observed, i.e. amplitude changes and high-frequency voltage ripples.

III. Power balance based model of the EAF

This paper presents an approach to EAF modeling with the application of chaos theory. The proposed model is based on a differential equation resulting from the power balance of the electric arc phenomenon. This equation, introduced in [24], is given by:

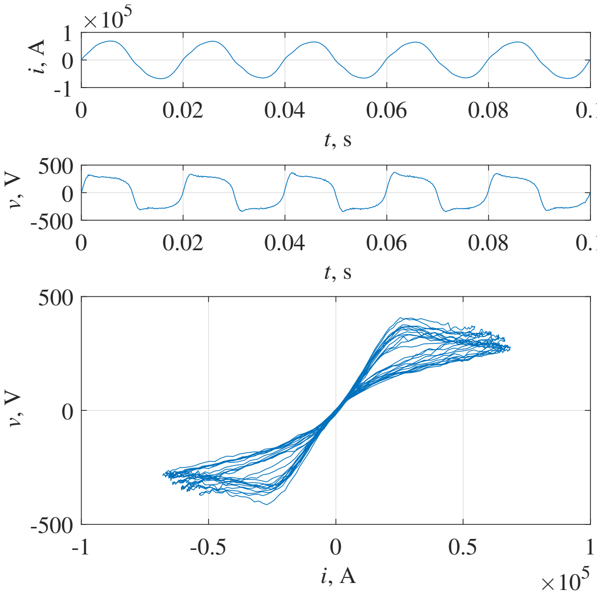

Fig. 2. Time waveforms and *v*-*i* characteristic of the single phase of the EAF recorded during the melting stage.

$$k_1 r^n(t) + k_2 r(t)\frac{\mathrm{d}r(t)}{\mathrm{d}t} = \frac{k_3}{r^{m+2}(t)} i^2(t), \tag{1}$$

where:

$r(t)$ – arc radius,
$i(t)$ – arc current,
k_j – proportionality coefficients, $j = 1, 2, 3$,
n, m – parameters, $n = 0, 1, 2$, $m = 0, 1, 2$,

while the arc voltage can be expressed as:

$$v(t) = \frac{k_3}{r^{m+2}(t)} i(t). \tag{2}$$

The parameters m and n are related to the stage of the EAF work cycle. For the melting stage, they take values of $m = 0$ and $n = 2$. Coefficients k_j are often considered constant. Exemplary values were given in [21] as $k_1 = 3000$, $k_2 = 1$, and $k_3 = 12.5$. The model expressed by (1) has been approved by the task force created by IEEE oriented towards dealing with simulation and modeling of harmonics [25] and is widely used.

A. Model's coefficient variability

As mentioned at the beginning of section III, the model coefficients k_j are usually assumed to be constant. The time-varying character of the model is then reflected by the addition of an appropriate component, e.g., in the form of a stochastic process. However, it can be assumed that these coefficients are specifically responsible for the variations in the EAF characteristic. That way, we rather assume their variability than constancy and consequently there is no need to add any new time-varying components. This approach was introduced in our previous research [26].

In this paper, we propose that the variability of k_j can be represented by signals generated by a chaotic system. Contrary to the results shown in [26], in this case the coefficients are deterministic.

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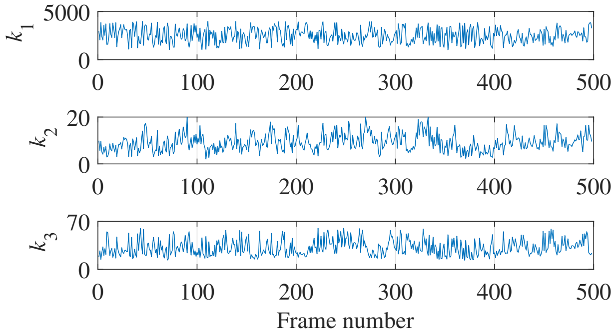

Fig. 3. Time sequences of k_j (j = 1, 2, 3) estimated by the genetic algorithm, based on measurement data.

The further analysis is based on the results of the estimation of the coefficients obtained with a genetic algorithm (GA). We have assumed that the coefficients remain constant for a period-long frame and then take a new value for the next period. First, the measured current and voltage waveforms have been divided into period-long frames, for which the GA has identified the model given by (1). The algorithm fitted coefficients k_j , minimizing the error between the model output and the measurement data. This resulted in the acquisition of three time sequences, one for each coefficient, which are presented in Fig. 3. An in-depth analysis of the estimation process was presented in [27].

IV. Modeling of the EAF using chaos theory

In this paper, we have assumed that the coefficients shown in Fig. 3 can be represented with waveforms obtained from a chaotic system. For this purpose, we have selected four systems. Three of them are widely known and often applied in engineering applications (including EAF modeling): Chua, Lorenz, and Rössler systems, and the fourth, which to our knowledge, have not yet been implemented in such a context.

In order to adjust the chaotic systems to the EAF modeling, some of their coefficients were fitted to the measurement data, while the other ones remained constant. A detailed description of the adjustment procedure is presented in subsection IV-E. It is important to emphasize that all coefficients were chosen in a way that ensures chaotic behavior.

A. Chua circuit

A simple autonomous circuit exhibiting chaotic behavior was first introduced in [28]. Since then, it has been applied in multiple engineering applications. The system can be described with a set of differential equations:

$$\begin{cases} \dot{x} = C_1(y - x - g(x)), \\ \dot{y} = x - y + z, \\ \dot{z} = -C_2 y, \\ g(x) = C_{d2}x + \dfrac{C_{d1} - C_{d2}}{2}(|x+1| - |x-1|), \end{cases} \tag{3}$$

where:

C_1, C_2 – coefficients related to resistance and capacitance of circuit elements,

C_{d1}, C_{d2} – parameters related to the slopes of the characteristic of the Chua diode.

Typical chaotic orbits examined in the case of the Chua system are obtained for $C_1 = 15.6$, $C_{d1} = -\frac{8}{7}$, $C_{d2} = -\frac{5}{7}$ and with variable C_2. In our case, the value C_2 was fitted to EAF-related data. To keep the signals chaotic, this coefficient can change in the following range $22.8 \leq C_2 \leq 33.6$. The variable y was chosen to represent the variability of the EAF coefficient k_j.

B. Lorenz system

The Lorenz chaotic system was introduced in [29] as a model related to unpredictability in weather. The set of equations can be formulated as follows:

$$\begin{cases} \dot{x} = L_1(y - x), \\ \dot{y} = x(L_2 - z) - y, \\ \dot{z} = xy - L_3 z, \end{cases} \tag{4}$$

where:

L_1 – coefficient related to the Prandtl number,

L_2 – coefficient related to the Rayleigh number,

L_3 – a geometric factor.

The dynamics of the Lorenz system is most often investigated for parameters $L_1 = 10$, $L_3 = \frac{8}{3}$, and for the variable L_2. The latter was fitted to the data, but in order to ensure chaotic behavior, it had to be greater than

$$L_2' = \frac{L_1(L_1 + L_3 + 3)}{L_1 - L_3 - 1} = 24.74. \tag{5}$$

The variable x was chosen to represent the variability of the EAF coefficient k_j.

C. Rössler system

The Rössler system, intended to behave similarly to the Lorenz system, was introduced in [30]. It is described with the following equations:

$$\begin{cases} \dot{x} = -(y + z), \\ \dot{y} = x + R_1 y, \\ \dot{z} = R_2 + z(x - R_3), \end{cases} \tag{6}$$

where:

R_1, R_2, R_3 – real-valued system coefficients.

The Rössler system was investigated, among others, for parameters $R_1 = R_2 = 0.2$ and variable R_3. We have assumed that R_3 was fitted in the limits $4.2 \leq R_3 \leq 8$. The variable y was chosen to represent the EAF coefficient k_j.

D. Four-wing chaotic attractor system

Based on the theoretical and practical experiences related to Chua, Lorenz, and Rössler systems the authors of [23] had introduced a new chaotic system. It exhibits a four-wing chaotic attractor with a very complicated topological structure over a wide range of parameters. The system is given by:

$$\begin{cases} \dot{x} = F_1(y - x) + F_5 yz, \\ \dot{y} = F_3 x + F_4 y - xz, \\ \dot{z} = -F_2 z + xy, \end{cases} \tag{7}$$

where:

F_1, F_2, F_4, F_5 – positive real-valued system coefficients,

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F_3 – real-valued coefficient.

The system was thoroughly investigated and described in the cited work [23]. The authors mainly deal with the simulation results obtained with $F_1 = 14$, $F_2 = 43$, $F_4 = 16$, $F_5 = 4$, and variable F_3. A wide range of chaotic behavior was observed for $-2.3 \leq F_3 \leq 3$. Similarly, in our case, the coefficients $F_1, F_2, F_4, and F_5$ were fixed as mentioned and F_3 was fitted to the data while maintaining the proposed range. The variable x was chosen to represent the EAF coefficient k_j.

E. Chaotic system fitting

In order to adjust the considered systems to the EAF data, we have applied a two-objective optimization process. The proposed goal functions are related to the distribution of the coefficient values k_j and their autocorrelation. Namely, they are designed to minimize the Cramér-von Mises distance between histograms and autocorrelation samples obtained from the chaotic system and measurement data. It should be noted that the distribution of the chaotic signal values is modified first, so that the median and kurtosis correspond to the distribution of the coefficient k_j. Formally, the goal functions can be written as:

$$
\begin{aligned}
f_1(\cdot) &= \sum_{p=1}^{N} (h_p^{meas} - h_p^{chaotic})^2, \\
f_2(\cdot) &= \sum_{p=1}^{M} (ACF_p^{meas} - ACF_p^{chaotic})^2,
\end{aligned}
\tag{8}
$$

where:

h_p – p-th histogram bar either from measurement or modified chaotic system data distribution,
N – number of histogram bars,
ACF_p – autocorrelation value for p-th lag,
M – number of lags for autocorrelation calculation.

The optimization process for each system is conducted three times, one for each coefficient k_j. In each optimization course and for each system, two variables are fitted: one is a coefficient occurring directly in the chaotic system equations from (3) to (7) and one related to the sampling frequency f_s. In the proposed EAF model, the realizations of k_j follow the results obtained from chaotic systems, but it is important to find an optimal sampling frequency which allows an accurate representation of k_j with a deterministic solution. Consequently, the goal functions take the following form:

$$
\begin{aligned}
&\min_{\xi, f_s} f_1(\xi, f_s), \\
&\min_{\xi, f_s} f_2(\xi, f_s),
\end{aligned}
\tag{9}
$$

where $\xi \in \{C_2, L_2, R_3, F_3\}$ represents an optimized parameter of the chaotic system.

Once the two-objective optimization is completed, a final solution is selected from a Pareto front as the closest to the ideal solution, which would be given by $f_1(\cdot) = 0$ and $f_2(\cdot) = 0$. More precisely, for each solution that belongs to the front, both values of the target function are normalized and then an Euclidean distance is calculated with respect to the ideal point $(0, 0)$. The best solution is characterized by the smallest computed distance.

V. Results of EAF Modeling

The optimization process described by the goal functions (9) was carried out separately for each chaotic system, as well as for each coefficient k_j. The models were designed in Matlab software and the computing infrastructure included a portable computer with Intel Core i7 processor (4 cores, 1.8 GHz), with 16 GB RAM and Windows 10 operating system. Table I presents the numerical results of the optimization process. It includes values of $\xi \in \{C_2, L_2, R_3, F_3\}$ and f_s fitted for every solution. Each table entry includes an error measure expressed as an Euclidean distance to the ideal solution, calculated on the basis of the normalized goal function values. The best solution is additionally highlighted with a bold font.

TABLE I
Results of the multi-objective optimization

		Chua	Lorenz	Rössler	Four-wing
	ξ	24.73	**41.00**	7.34	-0.28
k_1	f_s, S/s	0.18	**2.05**	0.43	3.00
	error	0.13	**0.08**	1.35	1.00
	ξ	24.15	27.13	7.86	**1.13**
k_2	f_s, S/s	0.30	3.40	0.45	**8.47**
	error	0.56	0.54	1.41	**0.47**
	ξ	24.21	**32.03**	7.85	-1.00
k_3	f_s, S/s	0.20	**2.48**	0.51	3.57
	error	0.62	**0.43**	1.41	0.59

The best solutions highlighted in Table I are also visualized to show their similarity to the measurement data. Figs. 4-6 present a comparison of the data used for the calculation of the goal function.As shown, the character of the k_j time sequence can be reflected by a chaotic system not only through the similar distribution of values but also their autocorrelation.

Based on the aforementioned results, we have proposed an EAF model in which k_1 and k_3 are represented with the help of the Lorenz system, while k_2 uses the four-wing attractor system. Fig. 7 presents a comparison between the measured voltage waveform and v-i characteristic of the EAF, and a single realization of the waveforms given by the proposed model. As shown, the model is capable of reflecting variations in the voltage waveform and v-i characteristic, especially in short-term frames. Nevertheless, some periods in which the shape distinctly differs from the other nearby periods can be rarely noticed, e.g., the higher voltage amplitude of the two periods visible in the v-i characteristic (Fig. 7).

VI. Conclusions

In this paper, we have presented an approach to EAF modeling using chaotic systems. It is based on a power balance equation that describes the electric arc phenomenon, where the coefficients are modeled by variables derived from a chaotic system. We have investigated and compared the application of four different chaotic systems. Each time, the chaotic system parameters have been fitted with the measurement data. Goal functions include a discrepancy between the distribution and autocorrelation of coefficient values.

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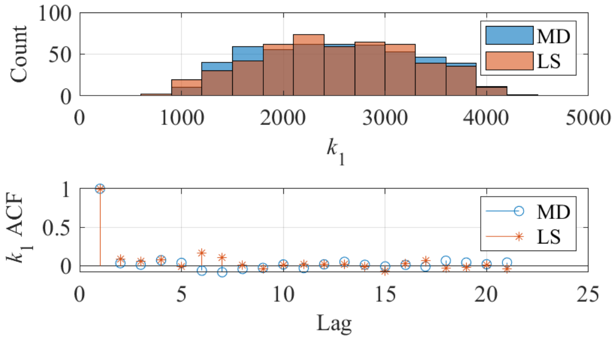

Fig. 4. Histogram and autocorrelation function of k_1 calculated on the base of measurement data (MD) and fitted using the Lorenz system (LS).

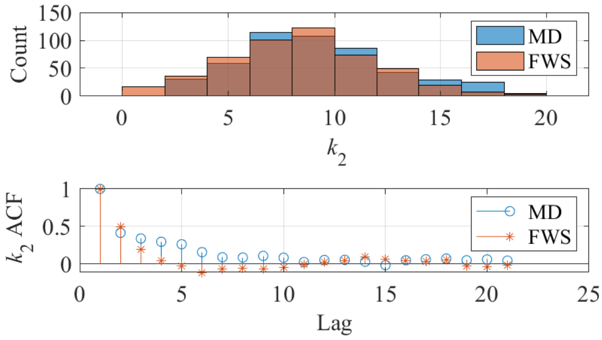

Fig. 5. Histogram and autocorrelation function of k_2 calculated on the base of measurement data (MD) and fitted using the four-wing attractor system (FWS).

The well-fitted parameters of a chaotic system, along with the appropriate sampling frequency, can ensure that the stochastic-like time sequences of the k_j coefficients are represented with an error small enough to simulate the output voltage properly. Such a model can then be easily implemented in software used for power system modeling. In that case a relation between voltage and current would be described by equations (1) and (2) with the k_j coefficients changing discretely in accordance with time sequences obtained from an appropriate chaotic system with optimized parameters. However, the proposed model is not free of drawbacks. Some

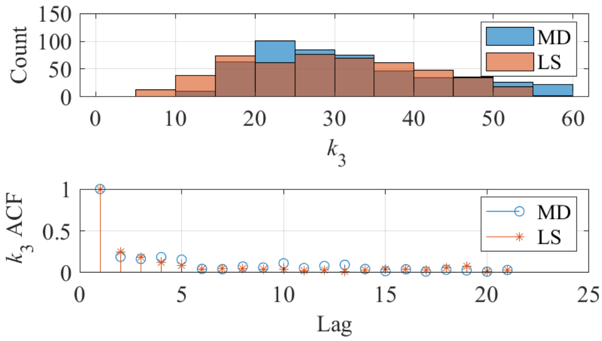

Fig. 6. Histogram and autocorrelation function of k_3 calculated on the base of measurement data (MD) and fitted using the Lorenz system (LS).

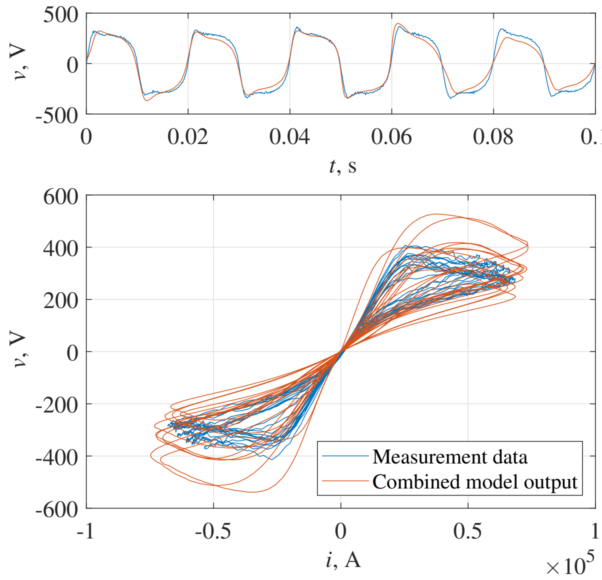

Fig. 7. Comparison of the measurement voltage waveform and v-i characteristic with a realization generated by a combined EAF model.

periods can be noticed in which the shape distinctly differs from the other nearby periods. Such a behavior results from the simplifying assumption that the k_j coefficients are not correlated with each other. An in-depth investigation of such correlations and their inclusion in the EAF model is planned for further research.

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