Two theorems about electromagnetic force in active anisotropic regions

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ABSTRACT

The paper has dealt with two problems of calculation of electromagnetic force/torque. The first one is for magnetically anisotropic and conductive region. It has been presented sufficient condition for surface-integral representation of electromagnetic force/torque in conductive and anisotropic region. The second approach deals with the problem of independence of force/torque calculated value from shape of integral-surface. The second theorem gives the sufficient condition for this independence for Maxwell stress tensor method is applied.

INTRODUCTION

This problem is analogous to the surface-integral representation of total electric charge placed in finite volume due to Gaussian law. It is known [1,2,4,5,6] that the surface-integral representation for electromagnetic field forces can be introduced for electromagnetic field regions if the Maxwell stress tensor is symmetrical. The symmetry of the Maxwell stress tensor is guaranteed for isotropic media, only [1,2]. The main interest is whether for some anisotropic media the surface-integral of force/torque representation is tenable. The answer is positive under an extended condition.

FIRST THEOREM

The first theorem considers the equivalence between both volume and surface integrals representations for total electromagnetic force/torque. Force density is given by equation

\[ \mathbf{f} = \mathbf{f}_v + \mathbf{N} + \mathbf{M} \]  \hspace{1cm} (1)

which is proved (see [12, Appendix]). The proof bases on Lorentz force density formula, Maxwells equations, and assumptions that displacement current (Poynting force [3]) and magnetic polarization (hysteresis force) can be neglected. In Eqs (1) electromagnetic field forces [1,2] theoretical analysis is still vital problem [3,4,5,6]. The anisotropy component is given as follows

\[ \mathbf{M} = \frac{1}{2} (\mathbf{v}_m - \mathbf{v}_w) \mathbf{B}_g \mathbf{grad}(\mathbf{B}_u) \]

for magnetic field region. Total electromagnetic force/torque can be calculated by the following equation

\[ \mathbf{f} = -\nabla \mathbf{u} \mathbf{v}_u + \mathbf{N} + \mathbf{M} \]

volume integral

surface integral

For isotropic media the reluctivity (permeability) matrix is diagonal and all prior values are equal to each other. The normal anisotropy is stated for the media which reluctivity (permeability) matrix is symmetrical one. There are also media for these the reluctivity (permeability) matrix is asymmetrical (the so-called anisotraic media [15]). The first theorem states:

**for magnetic field where the following conditions are satisfied**
- there is no nonhomogeneous force (no reluctance force),
- there is no hysteresis phenomenon appears,

the force calculated by Lorentz's (volume integral) and Maxwell (surface integral) methods lead to the same result for magnetically anisotropic region if reluctivity matrix is symmetrical i.e. for \( \mathbf{u} \neq \mathbf{v} \) it is satisfied

\[ \mathbf{V}_u = \mathbf{V}_w \]  \hspace{1cm} (4)

The mathematical proof of this theorem is based on Eqs (1) and the assumptions specified above. The Eqs (1) for \( u \)-th force component leads to the relation as follows

\[ f_u = f_{Lu} + M_u = -\nabla \mathbf{u} \mathbf{v}_u \]

For some coordinate system (Appendix) the residual component vanishes

\[ \Delta_u = 0 \]

thus

\[ L_u f_{Lu} = -\nabla \mathbf{u}(\mathbf{v}_u) \]

and finally

\[ \int f_v dV = \int \sigma_u dS \]

**SECOND THEOREM**

The electromagnetic force/torque value calculated with the help of either volume integral or Maxwell stress tensor surface integral does not depend on surface position in the gap if the condition (12) is satisfied

\[ \mathbf{V}_u \mathbf{v}_w = \mathbf{V}_w \mathbf{v}_u \]  \hspace{1cm} (12)

**EXAMPLE FOR FIRST & SECOND THEOREM**

Magnetic reluctivities

<table>
<thead>
<tr>
<th>The first theorem</th>
<th>The second theorem</th>
</tr>
</thead>
<tbody>
<tr>
<td>( v_{m} = 0.0\psi )</td>
<td>( v_{m} = 0.0\psi )</td>
</tr>
<tr>
<td>( v_{w} = 0.2\psi )</td>
<td>( v_{w} = 0.2\psi )</td>
</tr>
<tr>
<td>( v_{m} = 0.5\psi )</td>
<td>( v_{w} = 0.5\psi )</td>
</tr>
<tr>
<td>( v_{m} = 0 )</td>
<td>( v_{w} = 0 )</td>
</tr>
</tbody>
</table>

- \( (a) \) - condition (4) satisfied
- \( (b) \) - condition (4) satisfied
- \( (c) \) - condition (4) satisfied
- \( (d) \) - condition (4) satisfied

\[ T_{at} = \frac{1}{2}(v_m - v_w) \frac{dB}{c} dV \]

**REFERENCES**


ACKNOWLEDGEMENTS

There have been presented two theorems stated in the main text about electromagnetic force/torque in anisotropic media are proved by authors in 1997 and 2007.

**The first theorem**

for magnetic field where the following conditions are satisfied
- there is no nonhomogeneous force (no reluctance force),
- there is no hysteresis phenomenon appears,

the force calculated by Lorentz's (volume integral) and Maxwell (surface integral) methods lead to the same results for magnetically anisotropic region if reluctivity matrix is symmetrical i.e. for \( \mathbf{u} \neq \mathbf{v} \) it is satisfied

\[ \mathbf{V}_u = \mathbf{V}_w \]

**The second theorem**

The electromagnetic force/torque value calculated with the help of either volume integral or Maxwell stress tensor surface integral does not depend on surface position in the gap if the condition is satisfied

\[ \mathbf{V}_u \mathbf{v}_w = \mathbf{V}_w \mathbf{v}_u \]