Strategies for Optimal Allocation and Sizing of Active Power Filters

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***Abstract*— Some new approaches to active compensator allocation and sizing in distribution networks have been proposed in the paper. A few objective functions as well as their advantages and disadvantages have been given. Moreover, software which enables practical verification of the strategies has been described.**

Keywords-three phase systems; active power filters; power quality; compensator alloaction;

I. Introduction

The problem of waveform distortions in power systems could be solved with the help of additional passive or active compensators, e.g. active power filters (APF). In the past they have been usually selected individually. Now, the problem of compensator allocation and sizing becomes more important due to the more and more distributed character of distortion sources. Of course, the solution should ensure achievement of desired effects with the minimum technical and financial cost. Such broad approach to the problem seems to be essential while designing new supplying networks and managing the ones already in use if many loads causing voltage and current waveform distortions are present.

Optimization methods are widely used to solve problems in the field of power quality. Three basic groups of such problems can be pointed out:

- determination of compensator parameters [1],
- improving the efficiency of APF control algorithms [2], [3],
- allocation and sizing of compensators [4].

This paper deals only with the last of the above mentioned problems.

The optimal allocation of compensators has been considered both for passive [5], [6] and active filters [4], [7], [8], [9], [10], [11]. The aim of optimization usually consists in allocation of compensators having the minimum nominal currents which ensure distortion drop below the limits indicated by standards, e.g. [12], [13]. Because the nominal current influences the compensator price so the optimization leads also to cost reduction and the economic goal is automatically taken into account. The other approach consists in minimization of telephone interference factor (TIF) or voltage total harmonic distortion (THD) while keeping the compensator currents below specified values [4], [7], [14].

Impact of the load impedance or capacitor bank changes which influence the overall frequency spectra of distorted waveforms in system nodes is usually not considered with exception to few authors who assume that the analysis is made for the worst case [7], [15]. In fact, for some of the supplying systems the problem should be rather solved iteratively for successive steady states during the given time horizon. Sizing and allocation of compensators depends on many factors including the network structure, load patterns, location and characteristic of distortion sources. These factors are varying and should be in some way taken into account during the selection of compensators.

II. Propositions of Optimization Strategies

An objective function and some constraints need to be defined in order to get the optimal allocation of compensators used for higher harmonic suppression. In general, the set of independent (decision-making) variables includes compensator current phasors as well as nodes in which compensators are going to be installed.

It has been assumed that the power system under consideration is linearized – for each frequency nonlinear loads are modeled by current sources. Therefore, the system impedance matrix can be determined independently for each frequency. The compensation is carried out by means of APFs which are also modeled as current sources injecting higher harmonic currents to the system nodes, Fig. 1.

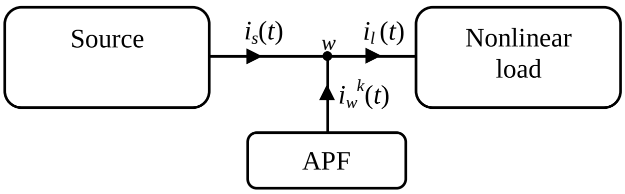

Figure 1. Block diagram of a system including APF.

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So an APF connected to a bus w can be described by Fourier series (the phase index has been omitted to simplify the notation, the superscript k denotes the compensator current):

$$i_w^k(t) = \sqrt{2}\,\mathrm{Re}\sum_{h=2}^{H} I_{wh}^k e^{jh\omega_0 t}\ , \qquad (1)$$

where:

H – maximum harmonic number,

I_{wh}^k – phasor of the h harmonic of the compensator current:

$$I_{wh}^k = \left|I_{wh}^k\right| e^{j\varphi_{wh}^k}\ , \qquad (2)$$

$\left|I_{wh}^k\right|$ – RMS value of the h harmonic,

φ_{wh}^k – phase of the h harmonic.

It has been assumed that the three phase system is symmetrical and phase currents are shifted by $\pm 2\pi/3$ copies of each other. In this case the compensator allocation can be based on analysis carried out for one of the phases. Otherwise, the proposed objective functions should be modified taking into account the need of symmetrization of minimum distortion level for each phase after connection of compensators [16]. This problem has not been considered in the paper.

A few new definitions of the objective function for the problem under consideration can be given. They are an extension of the basic definitions used in other works, e.g. [4], [9], [10]. The proposed objective functions can be combined. It leads to an objective function which is the most appropriate for the given problem of the compensator allocation and sizing.

The following sections (B-E) contain some new propositions of objective functions based on the standard approach described in section A.

A. Basic Objective Function

The basic objective function f_1 for the problem of compensator allocation is usually defined as the sum of RMS currents (which can take continuous or discrete values) [4], [8]:

$$\min_{\mathbf{x}} f_1(\mathbf{x}) = \min_{\left\{\mathrm{Re}(I_{wh}^k),\ \mathrm{Im}(I_{wh}^k)\right\}} \sum_{w=1}^{W} \left|I_w^k\right|, \qquad (3)$$

where:

W – number of buses to which compensators are attached ($W \le W'$, W' – total number of buses),

$\left|I_w^k\right|$ – RMS of the compensator current in the bus w:

$$\left|I_w^k\right| = \sqrt{\sum_{h=2}^{H}\left(\left(\mathrm{Re}(I_{wh}^k)\right)^2 + \left(\mathrm{Im}(I_{wh}^k)\right)^2\right)}\ . \qquad (4)$$

The problem (3) as well as all the following optimization problems are solved assuming all or some of the following constraints:

$$\left|I_w^k\right| - \left|I_w^k\right|_{\max} \le 0, \quad w = 1,2,..,W, \qquad (5)$$

$$\left|V_{wh}\right| - \left|V_h\right|_{\max} \le 0, \quad w = 1,2,..,W', \quad h = 2,3,\ldots,H, \qquad (6)$$

$$THDV_w - THDV_{\max} \le 0, \quad w = 1,2,..,W'. \qquad (7)$$

The first constraint is a result of the maximum acceptable compensator RMS current $\left|I_w^k\right|_{\max}$, the second one is a consequence of the maximum acceptable RMS of h order voltage harmonic $\left|V_h\right|_{\max}$ and the third one comes from the maximum value of the voltage THD coefficient ($THDV_{\max}$). Moreover, a lot of compensator manufacturers give the limit RMS values of successive current harmonics $\left|I_h^k\right|_{\max}$ what results in an additional constraint that must be fulfilled:

$$\left|I_{wh}^k\right| - \left|I_h^k\right|_{\max} \le 0, \quad w = 1,2,..,W. \qquad (8)$$

The problem (3) is very often solved iteratively for different numbers of compensators W. The starting point is usually W=W' and after the first step the compensators which do not have much influence on the solution are removed – the problem (3) is solved again for lower number of compensators [4], [7]. The procedure is repeated until the number of compensators reaches the minimum value which ensures fulfillment of the constraints (5) – (8).

The other approach consists in introducing additional decision-making variables β_w which can take values 0 and 1 representing the presence and the absence of a compensator connected to the bus w:

$$\min_{\mathbf{x}} f_2(\mathbf{x}) = \min_{\substack{\left\{\mathrm{Re}(I_{wh}^k),\ \mathrm{Im}(I_{wh}^k)\right\} \\ \{\beta_w\}}} \sum_{w=1}^{W} \beta_w \left|I_w^k\right|. \qquad (9)$$

Unfortunately, the discrete decision-making variables make it impossible to employ gradient optimization algorithms.

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B. *Cost Effective Objective Function*

The nonlinear transformation of the objective function f_1 leads to the following definition of the optimization problem:

$$\min_{\mathbf{x}} f_3(\mathbf{x}) = \min_{\left\{\mathrm{Re}(I^k_{wh}),\ \mathrm{Im}(I^k_{wh})\right\}} \sum_{w=1}^{W} g\left(\left|I^k_w\right|\right). \tag{10}$$

If the nonlinear function $g(\cdot)$ reflects relation between APF cost and its size (Fig. 2) then solution to the problem (10) leads to minimization of the economic cost. Unfortunately, in this case the function $g(\cdot)$ is not continuous and its step shape is a result of a discrete set of compensator ratings and depends on a company pricing policy. For example in Fig. 2 the set of compensator ratings is as follows: {100 A, 200 A, 300 A, 400 A, 500 A}. Solutions to the problem (10) for different functions $g(\cdot)$ may lead to different results. This approach can be applied to compare offers of a number of manufacturers.

The main disadvantage of using function f_3 comes from its discontinuity which results in impossibility of direct application of gradient optimization methods. This problem could be solved if the function $g(\cdot)$ is approximated by another continuous function, e.g. using splines, Fig. 3.

Another problem that could arise consists in having a few local minima with similar objective function values but obtained for different number of compensators.

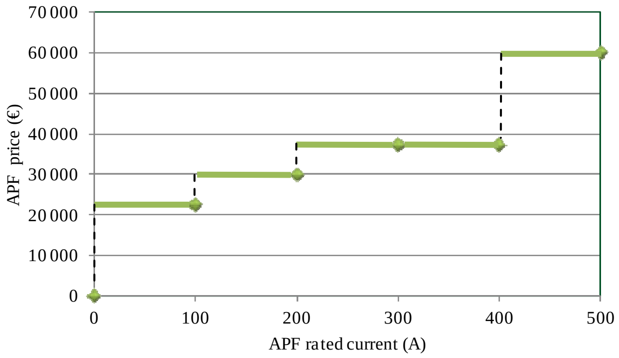

Figure 2. Exemplary function $g(\cdot)$ – relation between APF price and size.

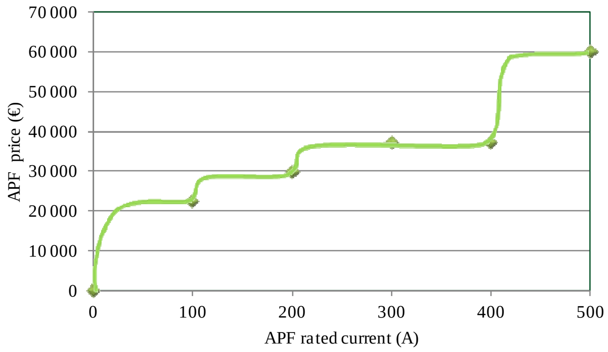

Figure 3. Continuous approximation of the function $g(\cdot)$.

In order to force solutions with less number of compensators a factor corresponding to fixed cost of installation and maintains for each APF can be added to the cost function g(·). In this case the function g(·) represents the dependence of the total cost including the purchase price and extra costs on the rated current and consequently solutions with minimum extra costs, i.e. less number of compensators, are preferred.

C. *Weighted Objective Function*

Sometimes some extra factors specific for each bus should be taken into account when solving the optimization problem (3). For example some buses could be preferred due to the accessibility or the simplicity of installation. It can be carried out by means of a multiplier $1/\alpha_w$ which depends on the bus number, Fig. 4:

$$\min_{\mathbf{x}} f_4(\mathbf{x}) = \min_{\left\{\mathrm{Re}(I^k_{wh}),\ \mathrm{Im}(I^k_{wh})\right\}} \sum_{w=1}^{W} \frac{1}{\alpha_w}\left|I^k_w\right|, \quad \alpha_w \in \langle 0{,}1\rangle. \tag{11}$$

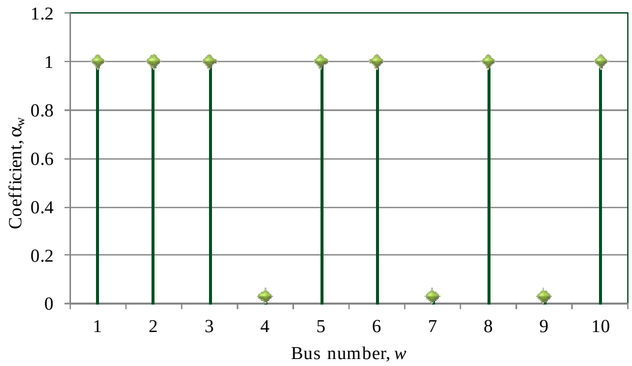

Figure 4. Exemplary set of coefficients α_w for a 10-bus system.

Optimization results for the objective functions f_1 and f_4 are equivalent if all the multipliers have the same value, particularly the value 1. The multipliers allow to prevent allocation of the compensators in selected buses for which $\alpha_w << 1$ (Fig. 4, buses #4, #7, #9) assuming that for the other buses $\alpha_w = 1$.

D. *Worst Case Objective Function*

The optimization problems considered so far lead to minimization of the objective functions being a sum of all compensator currents or a quality index defined on this sum. The problem under consideration can be also defined as a MinMax task. In such case the maximum compensator current is minimized:

$$\min_{\mathbf{x}} \max_{\mathbf{y}} f_5(\mathbf{x},\mathbf{y}) = \min_{w} \max_{\left\{\mathrm{Re}(I^k_{wh}),\ \mathrm{Im}(I^k_{wh})\right\}} \left|I^k_w\right|. \tag{12}$$

An exemplary difference in final results for both approaches has been presented in Fig. 5.

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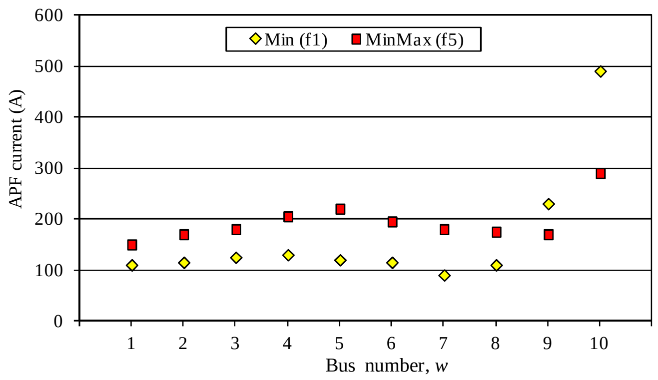

Figure 5. Exemple of APF currents obtained for Min and Minmax optimization strategies.

TABLE I. COMPARISON OF MIN AND MINMAX STRATEGIES

w	$\lvert I_w^k \rvert$, A		$\lvert I_w^k \rvert_{nom}$, A		Price, €	
	Min	*MinMax*	*Min*	*MinMax*	*Min*	*MinMax*
1	110	150	200	200	30 000	30 000
2	115	170	200	200	30 000	30 000
3	125	180	200	200	30 000	30 000
4	130	205	200	300	30 000	37 500
5	120	220	200	300	30 000	37 500
6	115	195	200	200	30 000	30 000
7	90	180	100	200	22 500	30 000
8	110	175	200	200	30 000	30 000
9	230	170	300	200	37 500	30 000
10	490	290	500	300	60 000	37 500
Σ	**1635**	**1935**	**2300**	**2300**	**330 000**	**322 500**

For the example shown in Fig. 5, the Min approach results in five APF sizes ranging from 100 A to 500 A, while for the MinMax only two sizes are required – 200 A and 300 A.

Table I presents the detailed comparison of the Min and MinMax strategies for the data shown in Fig. 5 and prices presented in Fig. 2. Although the total current consumed by compensators is less for the Min strategy (1635 A) than for the MinMax one (1935 A) it should be stressed that from the economical point of view the MinMax strategy leads to less investment costs (322 500 € comparing to 330 000 € for the Min). Of course, the results depend on the manufacturer pricing policy.

E. THDI Objective Function

The objective functions proposed in previous sections as well as the others which could be found for example in [7], [11], assume that the compensator can be regarded as a current source which can be freely adjustable within some range depending on the compensator rated harmonic currents. In fact, compensator control algorithms used contemporary allow rather to reduce local current distortions (THDI) and approach sinusoidal shape as close as possible in the bus under consideration. In order to follow this scheme the optimization problem should be defined as follows:

$$\min_{\mathbf{x}} f_6(\mathbf{x}) = \min_{\{\mathrm{Re}(I_{wh}^k),\, \mathrm{Im}(I_{wh}^k)\}} THDI_w, \quad w = 1,\ldots,W. \qquad (13)$$

In order to compare the basic strategy based on compensator currents (3) and the one described in this section a power system shown in Fig. 6 has been analyzed. It contains 20 buses with 8 DC distributed motors driven by 6-pulse line-commutated adjustable speed drives (ASD) which are main harmonic sources in the system [17].

The exemplary results have been obtained for the objective functions (3) and (13) assuming that a single APF is placed in the bus #12 (Apollo) which stands out because of the highest value of the voltage total harmonic distortion *THDV*. The sequential quadratic programming (SQP) algorithm implemented in Matlab has been applied to solve the optimization problems [18].

The objective function f_1 leads to the APF with higher nominal power comparing with the function f_6 but the final *THDV* values are lower for the function f_1 - see Table II.

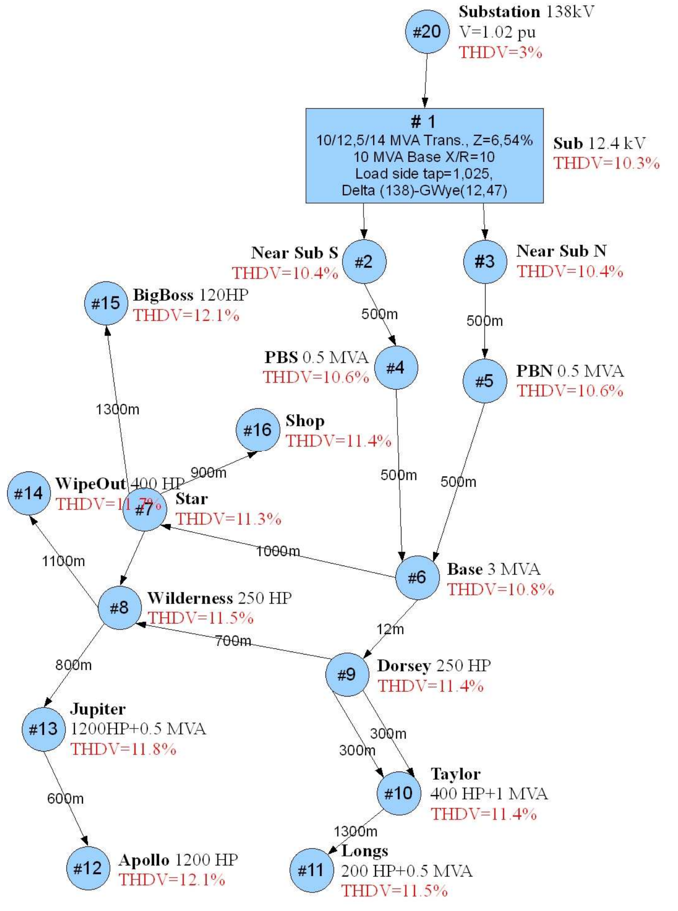

Figure 6. Test system diagram.

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TABLE II. COMPARISON OF OPTIMIZATION STRATEGIES

w	No APF		1 APF – f_1 (3)		1 APF – f_6 (13)	
	$THDV_w$ %	$THDI_w$ %	$THDV_w$ %	$THDI_w$ %	$THDV_w$ %	$THDI_w$ %
1	10.4	11.0	4.2	6.5	7.9	8.4
8	11.5	29.7	4.7	29.7	8.8	29.7
10	11.4	29.7	4.6	29.7	8.7	29.7
11	11.5	29.7	4.7	29.7	8.8	29.7
12	12.1	29.7	4.7	39.0	9.0	1.6
13	11.8	29.7	4.7	29.7	9.0	29.7
15	12.1	29.7	5.2	29.7	9.4	29.7
20	3.0	11.0	1.2	6.5	2.3	8.4

The objective function f_1 enables concurrent voltage distortion minimization at local and remote busses due to multi-point voltage monitoring but leads to more expensive solutions (higher APF ratings – Fig. 7) and what is more important it leads to increase of local current distortions (*THDI*) for the bus in which the APF has been installed – see Table II.

The problem with high values of *THDI* coefficients consists in that *THDI* limits the true power factor of nonlinear loads [17]. On the other hand using the objective function f_6 for a single APF does not allow to reach *THDV* values satisfying the standards [12], [13] although it leads to the smallest APF size and reduces the current distortion in the bus with the APF better than the other methods.

The APF current waveforms obtained using both optimization strategies have been shown in Fig. 7. The voltage and the line current waveforms for the bus #12 (Apollo) without APF and with APF have been presented in Figs. 8 – 10.

The current distortions in the bus #12 for the objective function f_6 (Fig. 10) are very small but the voltage distortions in this bus as well in the others exceed the limits. The current distortions in the bus #12 for the objective function f_1 (Fig. 9) are even higher than before optimization but the voltage distortions in this bus as well in the others are below the limits except for the bus #15 – see Table II.

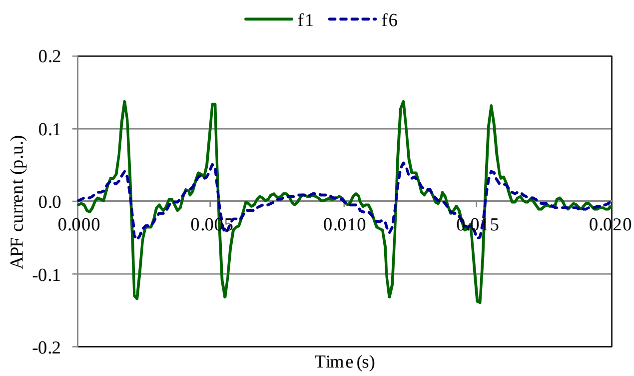

Figure 7. APF current waveforms for the objective function f_1 and f_6.

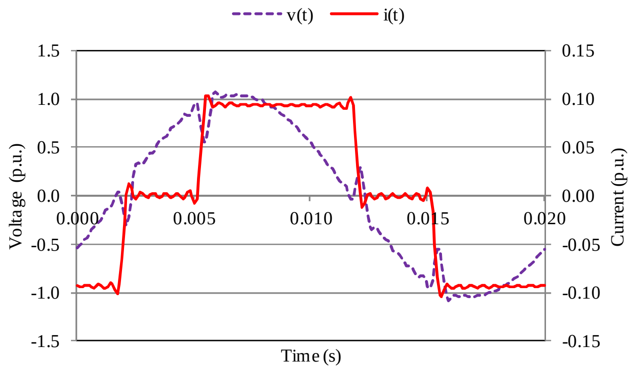

Figure 8. Voltage and current waveforms (bus #12) before compensation.

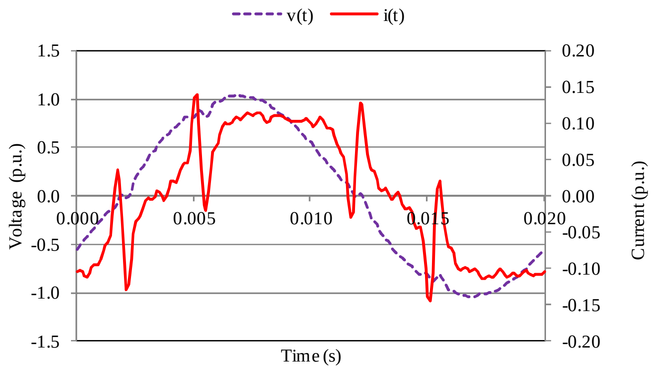

Figure 9. Voltage and current waveforms (bus #12) after compensation based on the objective function f_1.

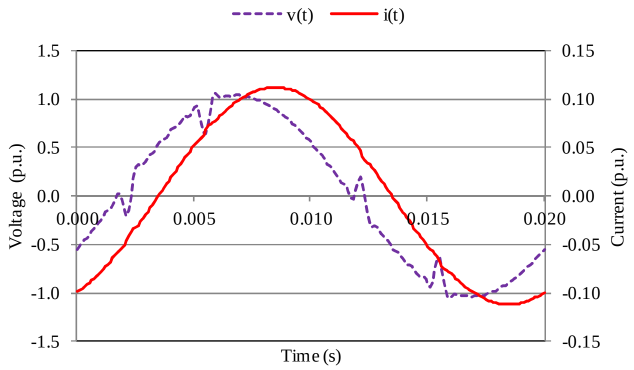

Figure 10. Voltage and current waveforms (bus #12) after compensation based on the objective function f_6.

III. APPLICATION OF PCFLO AND MATLAB TO SOLVE OPTIMAL APF ALLOCATION AND SIZING PROBLEMS

Determination of solutions to the optimization problems described in chapter II is carried out by means of:

- PCFLO [19] – software, which allows to analyze higher harmonics distribution in power systems,
- Matlab [18] – very powerful optimization algorithms,
- PcfloPackage [20] – library, which enables cooperation of PCFLO and Matlab.

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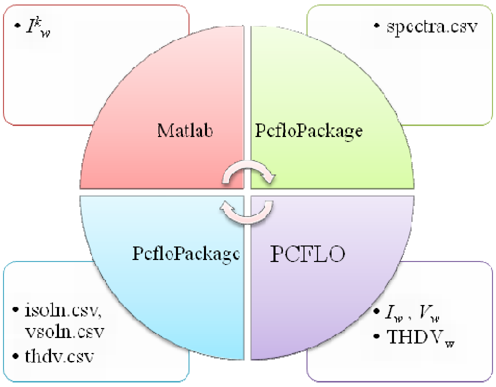

Figure 11. Information flow between Matlab and PCFLO during optimization process.

Information flow between software packages during solving the compensator allocation and sizing problems has been shown in Fig. 11.

Symbols in Fig. 11 denote phasors of bus currents I_w and voltages V_w, as well as voltage coefficients $THDV_w$. Files *.csv are used by PCFLO as input or output. They contain information about voltages (vsoln.csv), currents (isoln.csv), coefficients THDV (thdv.csv) and current sources used to model compensators (spectra.csv).

The range of optimization algorithms which can be used is very wide and depends among others on properties of the objective function, e.g. differentiability. First, the optimization problems of compensator allocation and sizing were solved with the help of GBDT algorithm [7] and combinatorial algorithms [21], later neural networks [8], TABU algorithm [11] and genetic algorithms [4], [9], [10] have been applied.

IV. Conclusions

Some strategies which enable optimal allocation and sizing of APFs in power systems have been proposed and compared in the paper. They consist in solving of optimization tasks. The sequential quadratic programming algorithm implemented in Matlab has been applied to solve these problems.

The successive steps of future works include application of other methods, especially evolutionary algorithms. The detailed comparative analysis of the proposed strategies for single and multiple APFs using several optimization algorithms should be the outcome of the future research work.

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