Estimation of the inner surface temperature of the opaque building components in a transient state using Exodus method

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Introduction

The research was carried out on the subject of the inner surface temperature estimation of the opaque building component (external wall). A Linear Time Invariant system was selected together with the Exodus simulation method. In order to verify if the proposed method is suitable, the measurements in the real object were performed and the results were compared with the output of the simulation.

Linear Time Invariant Systems

This paper focuses on a wall modelled as a Linear Time Invariant system with multiple inputs. For every such system there exists a function $\boldsymbol{h}$ such that the output is obtained as a convolution integral:

- in the continuous time:

$$y(t) = \int_{-\infty}^{\infty} h(\tau)x(t-\tau)d\tau \tag{1}$$

- in the discrete time:

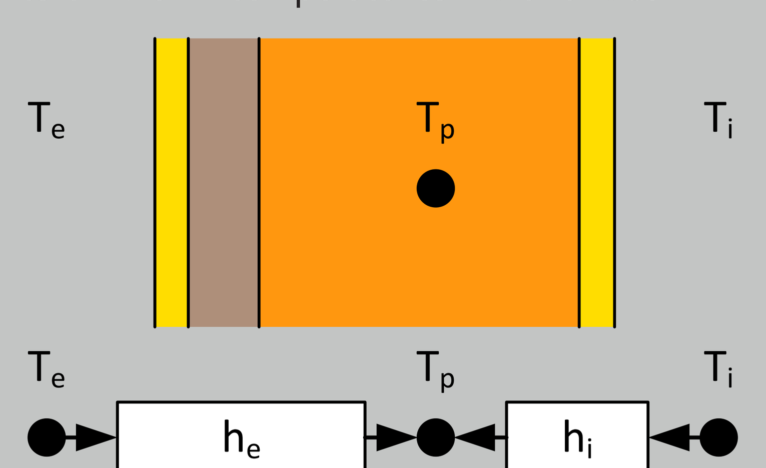
$$y(n) = \sum_{k=-\infty}^{\infty} h(k)x(n-k) \tag{2}$$

When the system has multiple inputs $\boldsymbol{S}$, the formula above can be generalized to the following form:

$$y(n) = \sum_{s}^{S} \sum_{k=-\infty}^{\infty} h_s(k)x_s(n-k) \tag{3}$$

Model of the wall

The analysis was related to an external wall built from multiple layers described by their: width, density, specific heat capacity and heat conductivity. It was surrounded by environments of known temperatures $\boldsymbol{Ti}$ and $\boldsymbol{Te}$.



Blocks labelled with hi and he represents subsystems with impulse response functions which define relation between the temperature of the point P and internal temperature and external temperature accordingly.

$$T_p(n) = \sum_{k=-\infty}^{\infty} h_i(k)T_i(n-k) + \sum_{k=-\infty}^{\infty} h_e(k)T_e(n-k) \tag{4}$$

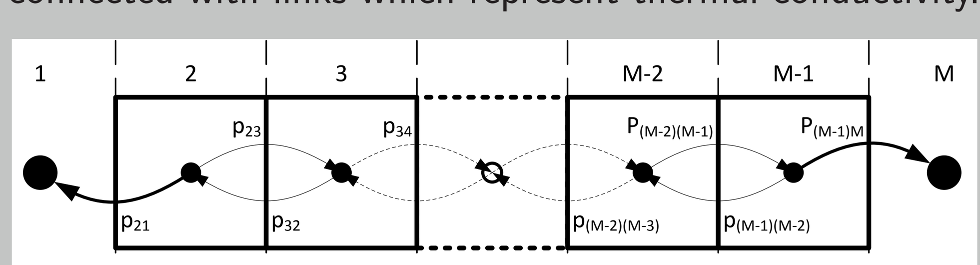
The Exodus method

The heat transfer through a wall can be simulated with the Exodus method. It is a modification of the Monte Carlo method. It was first introduced in 1968 by Emery and Carson.

It has the following properties:

- probabilistic in its approach,
- not subjected to randomness,
- no pseudo-random generator involved,
- lower computational complexity.

In order to use this method to a heat transfer problem the body needs to be divided into elements with a finite heat capacity. Then elements are converted into nodes of a mesh connected with links which represent thermal conductivity.



The mesh is filled with random walk probabilities obtained from forward difference quotient of the Poisson's equation.

$$T_{i,n+1} = p_{ii}T_{i,n} + \sum_{j}^{J_i} p_{ji}T_{j,n} + p_{iF}T_{F,n} \tag{5}$$

$$p_{ij} = \frac{\Delta t}{V_i c_p \varrho_i}\frac{1}{R_{ij}} \tag{6}$$

$$p_{iF} = \frac{\Delta t}{V_i c_p \varrho_i}\frac{1}{R_{iF}} \tag{7}$$

$$p_{ii} = 1 - \left(p_{iF} + \sum_{j}^{J_i} p_{ij}\right) \tag{8}$$

The Exodus method: execution

At the beginning of the exeution all particles are placed in the node for which the impulse response functions needs to be calculated. Their number is usually large (10^6). Then, in subsequent iterations particles move between nodes according to the random walk probabilities. If the particle reaches one of the pseudo nodes it is absorbed.

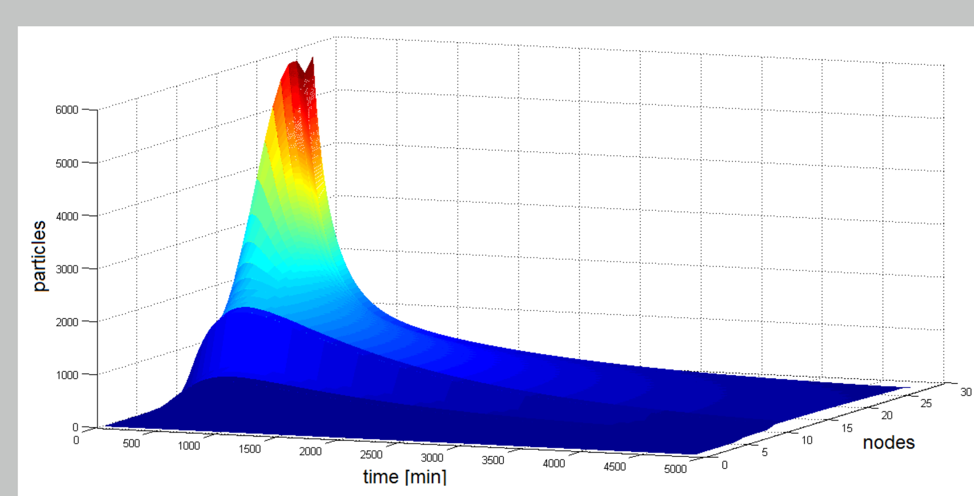
Values of the impulse response functions in the subsequent moments of time can be determined by the following formula:

$$h_a(n) = \frac{k_a n}{K} \tag{9}$$

where:
$h_a(n)$ - the value of the n^{th} sample of impulse response function for the environment $\boldsymbol{a}$,
k_{an} - number of the particles absorbed by the pseudo-node $\boldsymbol{a}$ in the n^{th} iteration,
$\boldsymbol{K}$ - initial number of particles.

The Exodus method: graphical interpretation

Because the execution can be viewed as a process of spreading particles in one dimensional space in time it can be presented in a tree-dimensional space.



Slices which run through boundary nodes correspond to the impulse response functions.

On-site measurements

Measurements were made in a single-family house in Silesia Region in Poland. The house was built in 1908 in a traditional solid brick. The physical parameters of different components were gathered in the following table.

Layer	λ $\left[\frac{W}{mK}\right]$	c_p $\left[\frac{J}{kgK}\right]$	ϱ $\left[\frac{kg}{m^3}\right]$	d $[mm]$
Plaster	0.82	840	1850	0.015
Solid brick	0.77	880	1800	0.38
Plaster	0.82	840	1850	0.015

In selected area (room), several temperatures were measured using thermocouples and multichannel recorder:

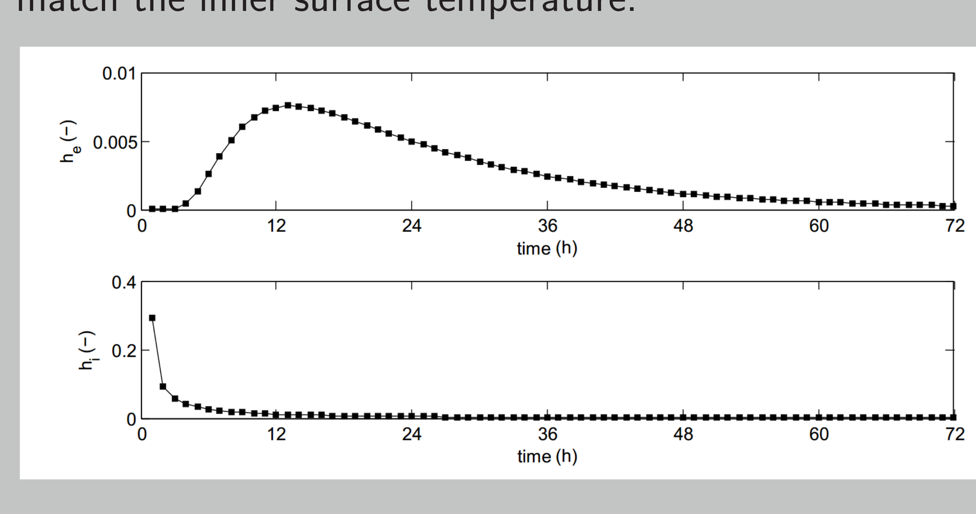
- external temperature,
- internal temperature,
- inner surface temperature.

Additionally, data from the nearby meteorological station was collected:

- total and diffusive solar radiation,
- wind speed.

Impulse response functions

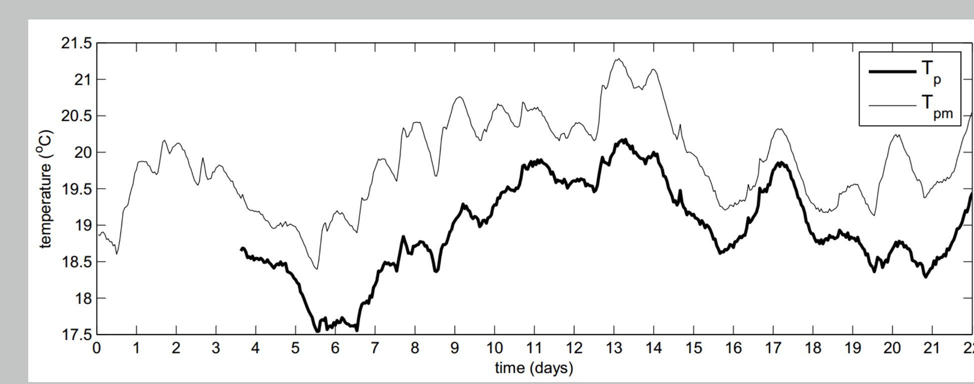
Although the method described can be used to produce a set of impulse response functions for an arbitrarily selected point within a wall, the research was focused on the first point from the inside. The expectation was that it will match the inner surface temperature.



- analyzed point was very close to the inner edge of the wall, thus the internal impulse response function $\boldsymbol{h_i}$ reaches much higher values than the external $\boldsymbol{h_e}$.
- the ambient temperature is initially meaningless and after three hours it starts to gain the importance,
- from fourteenth hour on, all functions slowly decrease to zero.

Initial results

Impulse response functions, external and internal temperatures were used to calculate the inner surface temperature $\boldsymbol{T_p}$ according to equation 4. The result was compared with the real $\boldsymbol{T_{pm}}$ measured.



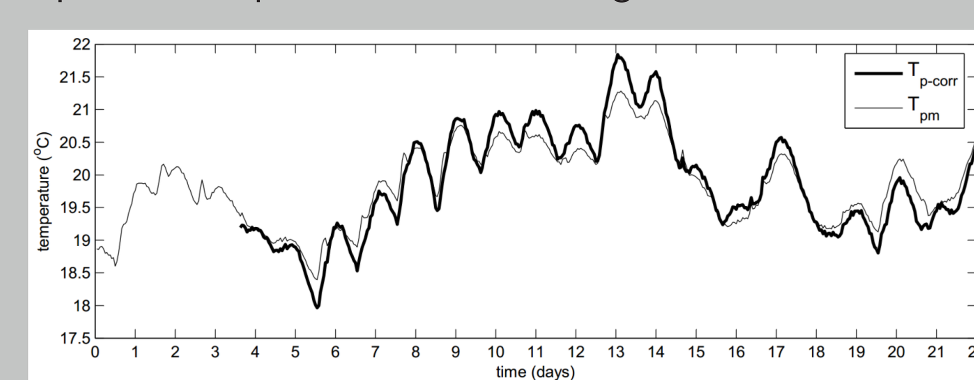
There was a discrepancy between result and the real measurements because initially the model did not include factors such as solar radiation and wind speed, which can have significant impact on the heat loss through the building envelope. Therefore, it was decided to look for opportunities to improve the model.

Sol-air temperature

The model was modified by replacing external air temperature with the sol-air temperature. It is defined as the hypothetical air temperature which compensates for the solar radiation heat gains.

$$T_{ec}(n) = T_e(n) + q_{s.z.}^{c.prom}(n) * a_{abs}^{s.z.} * R_e(n) \tag{10}$$

Sol-air temperature was use instead of the standard external temperature in the equation 4. The experiment was repeated and produced the following results.



The differences between the measured and calculated values during the measurement period were less than $\pm 0.6^o C$ and the maximum relative error reached 3%. The mean square error value was calculated as 0.0531. This result was considered to be satisfactory.

Conclusions

- The Exodus method can be used to model the heat transfer through the opaque building components and to obtain the temperature at any point inside the construction.
- The model is quite simple and does not take into account certain factors that may be relevant in the real world. They are: solar radiation and wind speed which affects the heat transfer coefficient. So it is necessary to include these parameters in the adjustment to the outdoor temperature by using the sol-air temperature.
- The modified model provides results very similar to the measured values, which gives hope that it will be possible to use this method in practice.
- The proposed method is a good alternative to measuring the inner surface temperature of the wall, as in the actual building it is difficult to perform, because of the need to mount (stick) of thermocouples in flats which are inhibited, and leaving there the expensive measuring equipment.
- The temperature of the inner surface of the wall which can be obtained using introduced method is particularly helpful in determining heat fluxes penetrating external walls of the building in transient conditions. It can be also used to obtain the average radiation temperature in the heated rooms in order to estimate the variable thermal comfort conditions. Moreover, the solution is applicable to verify the possibility of condensation of water vapor on the inner surface of the external wall.
- Although the Exodus method was first used in 1968, it is still applicable to many different areas, where stochastic approach fits best.

Future work

During the heating season 2012/2013 more measurements have been made for testing the presented algorithm and for validation of the model. The next stage provides the analysis of common technologies used for dwelling building in Poland in the last decades.

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